

On $(N(k), \xi)$ -semi-Riemannian manifolds: Pseudosymmetries

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Abstract. Definition of $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric semi-Riemannian manifold is given. $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified. Some results for \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are obtained. $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric semi-Riemannian manifolds are defined. $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified. Some results for $(R, \mathcal{T}_a, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are obtained. In particular, some results for (R, \mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are also obtained. After that, the definition of $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric semi-Riemannian manifold is given. $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified. It is proved that a $(R, S_{\mathcal{T}_a})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold is either Einstein or $L = k$ under an algebraic condition. Some results for (\mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are also obtained. In last, $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric semi-Riemannian manifolds are defined and $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified.

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1 Introduction

Let M be an n -dimensional differentiable manifold and $\mathfrak{X}(M)$ the Lie algebra of vector fields in M . We assume that $X, X_1, \dots, X_s, Y, Z, U, V, W \in \mathfrak{X}(M)$. It is well known that every $(1, 1)$ tensor field \mathcal{A} on a differentiable manifold determines a derivation $\mathcal{A} \cdot$ of the tensor algebra on the manifold, commuting with contractions. For example, a $(1, 1)$ tensor field $\mathcal{B}(V, U)$ induces the derivation $\mathcal{B}(V, U) \cdot$, thus given a $(0, s)$ tensor field \mathcal{K} , we can associate a $(0, s+2)$ tensor $\mathcal{B} \cdot \mathcal{K}$, defined by

$$\begin{aligned} (\mathcal{B} \cdot \mathcal{K})(X_1, \dots, X_s, X, Y) &= (\mathcal{B}(X, Y) \cdot \mathcal{K})(X_1, \dots, X_s) \\ &= -\mathcal{K}(\mathcal{B}(X, Y)X_1, \dots, X_s) - \dots \\ &\quad - \mathcal{K}(X_1, \dots, \mathcal{B}(X, Y)X_s). \end{aligned} \quad (1.1)$$

Next, for a tensor σ of type $(0, 2)$, the operators $(X \wedge_\sigma Y)$ and $Q(\sigma, \mathcal{K})$ are defined by

$$\begin{aligned} (X \wedge_\sigma Y)Z &= \sigma(Y, Z)X - \sigma(X, Z)Y, \\ Q(\sigma, \mathcal{K})(X_1, \dots, X_s, X, Y) &= ((X \wedge_\sigma Y) \cdot \mathcal{K})(X_1, \dots, X_s) \\ &= -\mathcal{K}((X \wedge_\sigma Y)X_1, \dots, X_s) - \dots \\ &\quad - \mathcal{K}(X_1, \dots, (X \wedge_\sigma Y)X_s). \end{aligned} \quad (1.2)$$

Let (M, g) be an n -dimensional semi-Riemannian manifold. Then (M, g) is said to be

(a) *pseudosymmetric* [6] if its curvature tensor R satisfies

$$R \cdot R = L_g Q(g, R),$$

where L_g is some smooth function on M ,

(b) *Ricci-generalized pseudosymmetric* [5] if it satisfies

$$R \cdot R = L_S Q(S, R),$$

where L_S is some smooth function on M and S is the the Ricci tensor,

(c) *Ricci-pseudosymmetric* [4] if on the set $\mathcal{U} = \{x \in M : (S - r/n)_x \neq 0\}$,

$$R \cdot S = LQ(g, S),$$

where L is some function on \mathcal{U} .

A pseudosymmetric manifold is a generalization of manifold of constant curvature, symmetric manifold ($\nabla R = 0$) and semisymmetric manifold ($R \cdot R = 0$). Deszcz et al. [7] proved that hypersurfaces in spaces of constant curvature, with exactly two distinct principal curvatures at every point, are pseudosymmetric. Ricci-pseudosymmetric manifold is a generalization of manifold of constant curvature, Einstein manifold, Ricci symmetric manifold ($\nabla S = 0$), symmetric manifold, semisymmetric manifold, pseudosymmetric manifold and Ricci-semisymmetric manifold ($R \cdot S = 0$). Similar to pseudosymmetry condition, Deszcz and Grycak [?, ?, ?] and Özgür [15] also studied Weyl pseudosymmetric manifolds ($R \cdot C = L_g Q(g, C)$). In 1990, Prvanović [21] studied the condition

$$R \cdot \tilde{T} = LQ(S^\ell, \tilde{T}), \quad \ell = 0, 1, 2, \dots,$$

where \tilde{T} is some $(0, 4)$ -tensor field and L is some smooth function on M . If $\tilde{T} = R$ and $\ell = 0$, this condition becomes the condition for pseudosymmetry and if $\tilde{T} = R$ and $\ell = 1$, this condition becomes the condition for Ricci-generalized pseudosymmetry.

Apart from curvature tensor and the Weyl conformal curvature tensor, quasi-conformal curvature tensor, concircular curavture tensor, conharmonic curvature tensor, pseudo-projective curvature tensor, projective curvature tensor are important curvature tensors in the semi-Riemannian point of view. It is interesting to study these curvature tensors and other curvature tensors on $(N(k), \xi)$ -semi-Riemannian manifolds. In this paper, we study several derivation conditions on $(N(k), \xi)$ -semi-Riemannian manifolds. The paper is organized as follows. In Section 2, we give the definition of \mathcal{T} -curvature tensor. In Section 3, we give examples and properties of $(N(k), \xi)$ -semi-Riemannian manifolds. In Section 4, $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric semi-Riemannian manifolds are defined and studied. Some results for \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are given. In Section 5, $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric semi-Riemannian manifolds are defined and studied. Some results for $(R, \mathcal{T}_a, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are given. In particular, some results for (R, \mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are obtained. In Section 6, the definition of $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric semi-Riemannian manifold is given. $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified. It is proved that a $(R, S_{\mathcal{T}_a})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold is either Einstein or $L = k$ under an algebraic condition. Some results for (\mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are obtained. In the last section, the definition of $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric semi-Riemannian manifold is given. $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifolds are classified.

2 Preliminaries

Definition 2.1 In an n -dimensional semi-Riemannian manifold (M, g) , \mathcal{T} -curvature tensor [27] is a tensor of type $(1, 3)$, which is defined by

$$\begin{aligned} \mathcal{T}(X, Y)Z &= a_0 R(X, Y)Z \\ &+ a_1 S(Y, Z)X + a_2 S(X, Z)Y + a_3 S(X, Y)Z \\ &+ a_4 g(Y, Z)QX + a_5 g(X, Z)QY + a_6 g(X, Y)QZ \\ &+ a_7 r(g(Y, Z)X - g(X, Z)Y), \end{aligned} \quad (2.1)$$

where a_0, \dots, a_7 are real numbers; and R, S, Q and r are the curvature tensor, the Ricci tensor, the Ricci operator and the scalar curvature respectively.

In particular, the \mathcal{T} -curvature tensor is reduced to

1. the *curvature tensor* R if

$$a_0 = 1, \quad a_1 = \cdots = a_7 = 0,$$

2. the *quasi-conformal curvature tensor* \mathcal{C}_* [31] if

$$a_1 = -a_2 = a_4 = -a_5, \quad a_3 = a_6 = 0, \quad a_7 = -\frac{1}{n} \left(\frac{a_0}{n-1} + 2a_1 \right),$$

3. the *conformal curvature tensor* \mathcal{C} [9, p. 90] if

$$a_0 = 1, \quad a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n-2}, \quad a_3 = a_6 = 0, \quad a_7 = \frac{1}{(n-1)(n-2)},$$

4. the *conharmonic curvature tensor* \mathcal{L} [10] if

$$a_0 = 1, \quad a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n-2}, \quad a_3 = a_6 = 0, \quad a_7 = 0,$$

5. the *concircular curvature tensor* \mathcal{V} ([29], [30, p. 87]) if

$$a_0 = 1, \quad a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0, \quad a_7 = -\frac{1}{n(n-1)},$$

6. the *pseudo-projective curvature tensor* \mathcal{P}_* [20] if

$$a_1 = -a_2, \quad a_3 = a_4 = a_5 = a_6 = 0, \quad a_7 = -\frac{1}{n} \left(\frac{a_0}{n-1} + a_1 \right),$$

7. the *projective curvature tensor* \mathcal{P} [30, p. 84] if

$$a_0 = 1, \quad a_1 = -a_2 = -\frac{1}{(n-1)}, \quad a_3 = a_4 = a_5 = a_6 = a_7 = 0,$$

8. the \mathcal{M} -*projective curvature tensor* [18] if

$$a_0 = 1, \quad a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2(n-1)}, \quad a_3 = a_6 = a_7 = 0,$$

9. the \mathcal{W}_0 -*curvature tensor* [18, Eq. (1.4)] if

$$a_0 = 1, \quad a_1 = -a_5 = -\frac{1}{(n-1)}, \quad a_2 = a_3 = a_4 = a_6 = a_7 = 0,$$

10. the \mathcal{W}_0^* -*curvature tensor* [18, Eq. (2.1)] if

$$a_0 = 1, \quad a_1 = -a_5 = \frac{1}{(n-1)}, \quad a_2 = a_3 = a_4 = a_6 = a_7 = 0,$$

11. the \mathcal{W}_1 -*curvature tensor* [18] if

$$a_0 = 1, \quad a_1 = -a_2 = \frac{1}{(n-1)}, \quad a_3 = a_4 = a_5 = a_6 = a_7 = 0,$$

12. the \mathcal{W}_1^* -*curvature tensor* [18] if

$$a_0 = 1, \quad a_1 = -a_2 = -\frac{1}{(n-1)}, \quad a_3 = a_4 = a_5 = a_6 = a_7 = 0,$$

13. the \mathcal{W}_2 -curvature tensor [17] if

$$a_0 = 1, \quad a_4 = -a_5 = -\frac{1}{(n-1)}, \quad a_1 = a_2 = a_3 = a_6 = a_7 = 0,$$

14. the \mathcal{W}_3 -curvature tensor [18] if

$$a_0 = 1, \quad a_2 = -a_4 = -\frac{1}{(n-1)}, \quad a_1 = a_3 = a_5 = a_6 = a_7 = 0,$$

15. the \mathcal{W}_4 -curvature tensor [18] if

$$a_0 = 1, \quad a_5 = -a_6 = \frac{1}{(n-1)}, \quad a_1 = a_2 = a_3 = a_4 = a_7 = 0,$$

16. the \mathcal{W}_5 -curvature tensor [19] if

$$a_0 = 1, \quad a_2 = -a_5 = -\frac{1}{(n-1)}, \quad a_1 = a_3 = a_4 = a_6 = a_7 = 0,$$

17. the \mathcal{W}_6 -curvature tensor [19] if

$$a_0 = 1, \quad a_1 = -a_6 = -\frac{1}{(n-1)}, \quad a_2 = a_3 = a_4 = a_5 = a_7 = 0,$$

18. the \mathcal{W}_7 -curvature tensor [19] if

$$a_0 = 1, \quad a_1 = -a_4 = -\frac{1}{(n-1)}, \quad a_2 = a_3 = a_5 = a_6 = a_7 = 0,$$

19. the \mathcal{W}_8 -curvature tensor [19] if

$$a_0 = 1, \quad a_1 = -a_3 = -\frac{1}{(n-1)}, \quad a_2 = a_4 = a_5 = a_6 = a_7 = 0,$$

20. the \mathcal{W}_9 -curvature tensor [19] if

$$a_0 = 1, \quad a_3 = -a_4 = \frac{1}{(n-1)}, \quad a_1 = a_2 = a_5 = a_6 = a_7 = 0.$$

Denoting

$$\mathcal{T}(X, Y, Z, V) = g(\mathcal{T}(X, Y)Z, V),$$

we write the curvature tensor \mathcal{T} in its $(0, 4)$ form as follows.

$$\begin{aligned} \mathcal{T}(X, Y, Z, V) = & a_0 R(X, Y, Z, V) \\ & + a_1 S(Y, Z)g(X, V) + a_2 S(X, Z)g(Y, V) \\ & + a_3 S(X, Y)g(Z, V) + a_4 S(X, V)g(Y, Z) \\ & + a_5 S(Y, V)g(X, Z) + a_6 S(Z, V)g(X, Y) \\ & + a_7 r(g(Y, Z)g(X, V) - g(X, Z)g(Y, V)). \end{aligned} \quad (2.2)$$

Notation 2.2 We will call \mathcal{T} -curvature tensor as \mathcal{T}_a -curvature tensor, whenever it is necessary. If a_0, \dots, a_7 are replaced by b_0, \dots, b_7 in the definition of \mathcal{T} -curvature tensor, then we will call \mathcal{T} -curvature tensor as \mathcal{T}_b -curvature tensor.

3 $(N(k), \xi)$ -semi-Riemannian manifolds

Let (M, g) be an n -dimensional semi-Riemannian manifold [14] equipped with a semi-Riemannian metric g . If $\text{index}(g) = 1$ then g is a Lorentzian metric and (M, g) a Lorentzian manifold [1]. If g is positive definite then g is an usual Riemannian metric and (M, g) a Riemannian manifold.

The k -nullity distribution [25] of (M, g) for a real number k is the distribution

$$N(k) : p \mapsto N_p(k) = \{Z \in T_p M : R(X, Y)Z = k(g(Y, Z)X - g(X, Z)Y)\}.$$

Let ξ be a non-null unit vector field in (M, g) and η its associated 1-form. Thus

$$g(\xi, \xi) = \varepsilon,$$

where $\varepsilon = 1$ or -1 according as ξ is spacelike or timelike, and

$$\eta(X) = \varepsilon g(X, \xi), \quad \eta(\xi) = 1. \quad (3.1)$$

Definition 3.1 An $(N(k), \xi)$ -semi-Riemannian manifold [28] consists of a semi-Riemannian manifold (M, g) , a k -nullity distribution $N(k)$ on (M, g) and a non-null unit vector field ξ in (M, g) belonging to $N(k)$.

Now, we intend to give some examples of $(N(k), \xi)$ -semi-Riemannian manifolds. For this purpose we collect some definitions from the geometry of almost contact manifolds and almost paracontact manifolds as follows:

Almost contact manifolds

Let M be a smooth manifold of dimension $n = 2m + 1$. Let φ , ξ and η be tensor fields of type $(1, 1)$, $(1, 0)$ and $(0, 1)$, respectively. If φ , ξ and η satisfy the conditions

$$\varphi^2 = -I + \eta \otimes \xi, \quad (3.2)$$

$$\eta(\xi) = 1, \quad (3.3)$$

where I denotes the identity transformation, then M is said to have an almost contact structure (φ, ξ, η) . A manifold M along with an almost contact structure is called an *almost contact manifold* [2]. Let g be a semi-Riemannian metric on M such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \varepsilon \eta(X) \eta(Y), \quad (3.4)$$

where $\varepsilon = \pm 1$. Then (M, g) is an (ε) -almost contact metric manifold [8] equipped with an (ε) -almost contact metric structure $(\varphi, \xi, \eta, g, \varepsilon)$. In particular, if the metric g is positive definite, then an (ε) -almost contact metric manifold is the usual *almost contact metric manifold* [2].

From (3.4), it follows that

$$g(X, \varphi Y) = -g(\varphi X, Y) \quad (3.5)$$

and

$$g(X, \xi) = \varepsilon \eta(X). \quad (3.6)$$

From (3.3) and (3.6), we have

$$g(\xi, \xi) = \varepsilon. \quad (3.7)$$

In an (ε) -almost contact metric manifold, the fundamental 2-form Φ is defined by

$$\Phi(X, Y) = g(X, \varphi Y). \quad (3.8)$$

An (ε) -almost contact metric manifold with $\Phi = d\eta$ is an (ε) -contact metric manifold [24]. For $\varepsilon = 1$ and g Riemannian, M is the usual *contact metric manifold* [2]. A contact metric manifold with $\xi \in N(k)$, is called a $N(k)$ -contact metric manifold [3].

An (ε) -almost contact metric structure $(\varphi, \xi, \eta, g, \varepsilon)$ is called an (ε) -Sasakian structure if

$$(\nabla_X \varphi) Y = g(X, Y) \xi - \varepsilon \eta(Y) X,$$

where ∇ is Levi-Civita connection with respect to the metric g . A manifold endowed with an (ε) -Sasakian structure is called an (ε) -Sasakian manifold [24]. For $\varepsilon = 1$ and g Riemannian, M is the usual Sasakian manifold [22, 2].

An almost contact metric manifold is a Kenmotsu manifold [12] if

$$(\nabla_X \varphi) Y = g(\varphi X, Y) \xi - \eta(Y) \varphi X. \quad (3.9)$$

By (3.9), we have

$$\nabla_X \xi = X - \eta(X) \xi. \quad (3.10)$$

Almost paracontact manifolds

Let M be an n -dimensional almost paracontact manifold [23] equipped with an almost paracontact structure (φ, ξ, η) , where φ , ξ and η are tensor fields of type $(1, 1)$, $(1, 0)$ and $(0, 1)$, respectively; and satisfy the conditions

$$\varphi^2 = I - \eta \otimes \xi, \quad (3.11)$$

$$\eta(\xi) = 1. \quad (3.12)$$

Let g be a semi-Riemannian metric on M such that

$$g(\varphi X, \varphi Y) = g(X, Y) - \varepsilon \eta(X) \eta(Y), \quad (3.13)$$

where $\varepsilon = \pm 1$. Then (M, g) is an (ε) -almost paracontact metric manifold equipped with an (ε) -almost paracontact metric structure $(\varphi, \xi, \eta, g, \varepsilon)$. In particular, if $\text{index}(g) = 1$, then an (ε) -almost paracontact metric manifold is said to be a Lorentzian almost paracontact manifold. In particular, if the metric g is positive definite, then an (ε) -almost paracontact metric manifold is the usual almost paracontact metric manifold [23].

The equation (3.13) is equivalent to

$$g(X, \varphi Y) = g(\varphi X, Y) \quad (3.14)$$

along with

$$g(X, \xi) = \varepsilon \eta(X). \quad (3.15)$$

From (3.12) and (3.15), we have

$$g(\xi, \xi) = \varepsilon. \quad (3.16)$$

An (ε) -almost paracontact metric structure is called an (ε) -para-Sasakian structure [26] if

$$(\nabla_X \varphi) Y = -g(\varphi X, \varphi Y) \xi - \varepsilon \eta(Y) \varphi^2 X, \quad (3.17)$$

where ∇ is Levi-Civita connection with respect to the metric g . A manifold endowed with an (ε) -para-Sasakian structure is called an (ε) -para-Sasakian manifold [26]. For $\varepsilon = 1$ and g Riemannian, M is the usual para-Sasakian manifold [23]. For $\varepsilon = -1$, g Lorentzian and ξ replaced by $-\xi$, M becomes a Lorentzian para-Sasakian manifold [13].

Example 3.2 [28] *The following are some well known examples of $(N(k), \xi)$ -semi-Riemannian manifolds:*

1. An $N(k)$ -contact metric manifold [3] is an $(N(k), \xi)$ -Riemannian manifold.
2. A Sasakian manifold [22] is an $(N(1), \xi)$ -Riemannian manifold.
3. A Kenmotsu manifold [12] is an $(N(-1), \xi)$ -Riemannian manifold.
4. An (ε) -Sasakian manifold [24] an $(N(\varepsilon), \xi)$ -semi-Riemannian manifold.

5. A para-Sasakian manifold [23] is an $(N(-1), \xi)$ -Riemannian manifold.

6. An (ε) -para-Sasakian manifold [26] is an $(N(-\varepsilon), \xi)$ -semi-Riemannian manifold.

In an n -dimensional $(N(k), \xi)$ -semi-Riemannian manifold (M, g) , it is easy to verify that

$$R(X, Y)\xi = \varepsilon k(\eta(Y)X - \eta(X)Y), \quad (3.18)$$

$$R(\xi, X)Y = \varepsilon k(\varepsilon g(X, Y)\xi - \eta(Y)X), \quad (3.19)$$

$$R(\xi, X)\xi = \varepsilon k(\eta(X)\xi - X), \quad (3.20)$$

$$R(X, Y, Z, \xi) = \varepsilon k(\eta(X)g(Y, Z) - \eta(Y)g(X, Z)), \quad (3.21)$$

$$\eta(R(X, Y)Z) = k(\eta(X)g(Y, Z) - \eta(Y)g(X, Z)), \quad (3.22)$$

$$S(X, \xi) = \varepsilon k(n-1)\eta(X), \quad (3.23)$$

$$Q\xi = k(n-1)\xi, \quad (3.24)$$

$$S(\xi, \xi) = \varepsilon k(n-1), \quad (3.25)$$

$$\eta(QX) = \varepsilon g(QX, \xi) = \varepsilon S(X, \xi) = k(n-1)\eta(X). \quad (3.26)$$

Moreover, define

$$S^\ell(X, Y) = g(Q^\ell X, Y) = S(Q^{\ell-1}X, Y), \quad (3.27)$$

where $\ell = 0, 1, 2, \dots$ and $S^0 = g$. Using (3.26) in (3.27), we get

$$S^\ell(X, \xi) = \varepsilon k^\ell(n-1)^\ell \eta(X). \quad (3.28)$$

Now, we give the following Lemma.

Lemma 3.3 [28] *Let M be an n -dimensional $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned} \mathcal{T}_a(X, Y)\xi &= (-\varepsilon k a_0 + \varepsilon k(n-1)a_2 - \varepsilon a_7 r)\eta(X)Y \\ &\quad + (\varepsilon k a_0 + \varepsilon k(n-1)a_1 + \varepsilon a_7 r)\eta(Y)X \\ &\quad + a_3 S(X, Y)\xi + \varepsilon a_4 \eta(Y)QX \\ &\quad + \varepsilon a_5 \eta(X)QY + k(n-1)a_6 g(X, Y)\xi, \end{aligned} \quad (3.29)$$

$$\begin{aligned} \mathcal{T}_a(\xi, X)\xi &= (-\varepsilon k a_0 + \varepsilon k(n-1)a_2 - \varepsilon a_7 r)X + \varepsilon a_5 QX \\ &\quad + \{\varepsilon k a_0 + \varepsilon k(n-1)a_1 + \varepsilon k(n-1)a_3 \\ &\quad + \varepsilon k(n-1)a_4 + \varepsilon k(n-1)a_6 + \varepsilon a_7 r\}\eta(X)\xi, \end{aligned} \quad (3.30)$$

$$\begin{aligned} \mathcal{T}_a(\xi, Y)Z &= (k a_0 + k(n-1)a_4 + a_7 r)g(Y, Z)\xi \\ &\quad + a_1 S(Y, Z)\xi + \varepsilon k(n-1)a_3 \eta(Y)Z \\ &\quad + \varepsilon a_5 \eta(Z)QY + \varepsilon a_6 \eta(Y)QZ \\ &\quad + (-\varepsilon k a_0 + \varepsilon k(n-1)a_2 - \varepsilon a_7 r)\eta(Z)Y, \end{aligned} \quad (3.31)$$

$$\begin{aligned} \eta(\mathcal{T}_a(X, Y)\xi) &= \varepsilon k(n-1)(a_1 + a_2 + a_4 + a_5)\eta(X)\eta(Y) \\ &\quad + a_3 S(X, Y) + k(n-1)a_6 g(X, Y), \end{aligned} \quad (3.32)$$

$$\begin{aligned} \mathcal{T}_a(X, Y, \xi, V) &= (-\varepsilon k a_0 + \varepsilon k(n-1)a_2 - \varepsilon a_7 r)\eta(X)g(Y, V) \\ &\quad + (\varepsilon k a_0 + \varepsilon k(n-1)a_1 + \varepsilon a_7 r)\eta(Y)g(X, V) \\ &\quad + \varepsilon a_3 S(X, Y)\eta(V) + \varepsilon a_4 \eta(Y)S(X, V) \\ &\quad + \varepsilon a_5 \eta(X)S(Y, V) + \varepsilon k(n-1)a_6 g(X, Y)\eta(V), \end{aligned} \quad (3.33)$$

$$\begin{aligned}
\mathcal{T}_a(X, \xi)\xi &= \{-\varepsilon k a_0 + \varepsilon k(n-1)a_2 + \varepsilon k(n-1)a_3 \\
&\quad + \varepsilon k(n-1)a_5 + \varepsilon k(n-1)a_6 - \varepsilon a_7 r\} \eta(X)\xi \\
&\quad + (\varepsilon k a_0 + \varepsilon k(n-1)a_1 + \varepsilon a_7 r) X + \varepsilon a_4 QX,
\end{aligned} \tag{3.34}$$

$$\begin{aligned}
S_{\mathcal{T}_a}(X, \xi) &= \{\varepsilon k(n-1)(a_0 + n a_1 + a_2 + a_3 + a_5 + a_6) \\
&\quad + \varepsilon r(a_4 + (n-1)a_7)\} \eta(X),
\end{aligned} \tag{3.35}$$

$$\begin{aligned}
S_{\mathcal{T}_a}(\xi, \xi) &= \varepsilon k(n-1)(a_0 + n a_1 + a_2 + a_3 + a_5 + a_6) \\
&\quad + \varepsilon r(a_4 + (n-1)a_7).
\end{aligned} \tag{3.36}$$

Remark 3.4 *The relations (3.18) – (3.36) are true for*

1. *a $N(k)$ -contact metric manifold [3] ($\varepsilon = 1$),*
2. *a Sasakian manifold [22] ($k = 1, \varepsilon = 1$),*
3. *a Kenmotsu manifold [12] ($k = -1, \varepsilon = 1$),*
4. *an (ε) -Sasakian manifold [24] ($k = \varepsilon, \varepsilon k = 1$),*
5. *a para-Sasakian manifold [23] ($k = -1, \varepsilon = 1$), and*
6. *an (ε) -para-Sasakian manifold [26] ($k = -\varepsilon, \varepsilon k = -1$).*

Even, all the relations and results of this paper will be true for the above six cases.

4 $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetry

Definition 4.1 A semi-Riemannian manifold (M, g) is said to be $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric if

$$\mathcal{T}_a \cdot \mathcal{T}_b = L_g Q(g, \mathcal{T}_b), \tag{4.1}$$

where L_g is some smooth function on M . In particular, it is said to be (R, \mathcal{T}_a) -pseudosymmetric or, in brief, \mathcal{T}_a -pseudosymmetric if

$$R \cdot \mathcal{T}_a = L_g Q(g, \mathcal{T}_a) \tag{4.2}$$

holds on the set

$$\mathcal{U}_g = \{x \in M : (\mathcal{T}_a)_x \neq 0 \text{ at } x\},$$

where L_g is some smooth function on \mathcal{U}_g . In particular, if in (4.2), \mathcal{T}_a is equal to $R, \mathcal{C}_*, \mathcal{C}, \mathcal{L}, \mathcal{V}, \mathcal{P}_*, \mathcal{P}, \mathcal{M}, \mathcal{W}_0, \mathcal{W}_0^*, \mathcal{W}_1, \mathcal{W}_1^*, \mathcal{W}_2, \mathcal{W}_3, \mathcal{W}_4, \mathcal{W}_5, \mathcal{W}_6, \mathcal{W}_7, \mathcal{W}_8, \mathcal{W}_9$, then it becomes pseudosymmetric, quasi-conformal pseudosymmetric, Weyl pseudosymmetric, conharmonic pseudosymmetric, concircular pseudosymmetric, pseudo-projective pseudosymmetric, projective pseudosymmetric, \mathcal{M} -pseudosymmetric, \mathcal{W}_0 -pseudosymmetric, \mathcal{W}_0^* -pseudosymmetric, \mathcal{W}_1 -pseudosymmetric, \mathcal{W}_1^* -pseudosymmetric, \mathcal{W}_2 -pseudosymmetric, \mathcal{W}_3 -pseudosymmetric, \mathcal{W}_4 -pseudosymmetric, \mathcal{W}_5 -pseudosymmetric, \mathcal{W}_6 -pseudosymmetric, \mathcal{W}_7 -pseudosymmetric, \mathcal{W}_8 -pseudosymmetric, \mathcal{W}_9 -pseudosymmetric, respectively.

Theorem 4.2 *Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& \varepsilon b_0(ka_0 + \varepsilon k(n-1)a_4 + a_7r)R(U, V, W, X) + \varepsilon a_1b_0S(X, R(U, V)W) \\
& - 2k(n-1)a_3(kb_0 + k(n-1)b_4 + b_7r)\eta(X)\eta(U)g(V, W) \\
& - 2k(n-1)a_3(-kb_0 + k(n-1)b_5 - b_7r)\eta(X)\eta(V)g(U, W) \\
& + \varepsilon a_1b_4S^2(X, U)g(V, W) + \varepsilon a_1b_5S^2(X, V)g(U, W) \\
& + \varepsilon a_1b_6S^2(X, W)g(U, V) - a_5(b_1 + b_3)S^2(X, V)\eta(U)\eta(W) \\
& - a_5(b_1 + b_2)S^2(X, W)\eta(U)\eta(V) - a_5(b_2 + b_3)S^2(X, U)\eta(V)\eta(W) \\
& - 2a_6b_1S^2(V, W)\eta(X)\eta(U) - 2a_6b_2S^2(U, W)\eta(X)\eta(V) \\
& - 2a_6b_3S^2(U, V)\eta(X)\eta(W) - 2k^2(n-1)a_3b_6g(U, V)\eta(X)\eta(W) \\
& - 2(k(n-1)a_3b_1 + a_6(kb_0 + k(n-1)b_4 + b_7r))\eta(X)\eta(U)S(V, W) \\
& - 2(k(n-1)a_3b_2 + a_6(-kb_0 + k(n-1)b_5 - b_7r))\eta(X)\eta(V)S(U, W) \\
& - 2k(n-1)(a_3b_3 + a_6b_6)S(U, V)\eta(X)\eta(W) \\
& + \varepsilon(b_4(ka_0 + k(n-1)a_4 + a_7r) - a_1(kb_0 + k(n-1)b_4))S(X, U)g(V, W) \\
& + \varepsilon(b_5(ka_0 + k(n-1)a_4 + a_7r) - a_1(-kb_0 + k(n-1)b_5))S(X, V)g(U, W) \\
& + \varepsilon b_6(ka_0 + k(n-1)(a_4 - a_1) + a_7r)S(X, W)g(U, V) \\
& - \varepsilon(kb_0 + k(n-1)b_4)(ka_0 + k(n-1)a_4 + a_7r)g(X, U)g(V, W) \\
& - \varepsilon(-kb_0 + k(n-1)b_5)(ka_0 + k(n-1)a_4 + a_7r)g(U, W)g(X, V) \\
& - \varepsilon k(n-1)b_6(ka_0 + k(n-1)a_4 + a_7r)g(X, W)g(U, V) \\
& - k(n-1)((b_2 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(-kb_0 + k(n-1)(b_5 + b_6) - b_7r))g(X, U)\eta(V)\eta(W) \\
& - k(n-1)((b_1 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))g(X, V)\eta(U)\eta(W) \\
& - ((b_1 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))S(X, V)\eta(U)\eta(W) \\
& - ((b_2 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_5 + b_6) + b_7r))S(X, U)\eta(V)\eta(W) \\
& - ((b_1 + b_2)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + k(n-1)(b_4 + b_5)(a_1 + a_5))S(X, W)\eta(U)\eta(V) \\
& - k(n-1)(k(n-1)(b_4 + b_5)(a_2 + a_4) \\
& + (b_1 + b_2)(ka_0 + k(n-1)a_4 + a_7r))g(X, W)\eta(U)\eta(V) \\
= & L_g(\varepsilon b_0R(U, V, W, X) + \varepsilon b_4S(X, U)g(V, W) + \varepsilon b_5S(X, V)g(U, W) \\
& + \varepsilon b_6S(X, W)g(U, V) - \varepsilon k(n-1)b_6g(U, V)g(X, W) \\
& - k(n-1)(b_2 + b_3)g(X, U)\eta(V)\eta(W) \\
& - k(n-1)(b_1 + b_3)g(X, V)\eta(U)\eta(W) \\
& - k(n-1)(b_1 + b_2)g(X, W)\eta(U)\eta(V) \\
& + (b_2 + b_3)S(X, U)\eta(V)\eta(W) \\
& + (b_1 + b_3)S(X, V)\eta(U)\eta(W) \\
& + (b_1 + b_2)S(X, W)\eta(U)\eta(V) \\
& - \varepsilon(-kb_0 + k(n-1)b_5)g(U, W)g(X, V) \\
& - \varepsilon(kb_0 + k(n-1)b_4)g(V, W)g(X, U)). \tag{4.3}
\end{aligned}$$

Proof. Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\mathcal{T}_a(Z, X) \cdot \mathcal{T}_b(U, V)W = L_g Q(g, \mathcal{T}_b)(U, V, W; Z, X). \tag{4.4}$$

Taking $Z = \xi$ in (4.4), we get

$$\mathcal{T}_a(\xi, X) \cdot \mathcal{T}_b(U, V)W = L_g Q(g, \mathcal{T}_b)(U, V, W; \xi, X),$$

which gives

$$\begin{aligned} & \mathcal{T}_a(\xi, X)\mathcal{T}_b(U, V)W - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V)W \\ & - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V)W - \mathcal{T}_b(U, V)\mathcal{T}_a(\xi, X)W \\ = & L_g((\xi \wedge X)\mathcal{T}_b(U, V)W - \mathcal{T}_b((\xi \wedge X)U, V)W \\ & - \mathcal{T}_b(U, (\xi \wedge X)V)W - \mathcal{T}_b(U, V)(\xi \wedge X)W), \end{aligned}$$

that is,

$$\begin{aligned} & \mathcal{T}_a(\xi, X)\mathcal{T}_b(U, V)W - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V)W \\ & - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V)W - \mathcal{T}_b(U, V)\mathcal{T}_a(\xi, X)W \\ = & L_g(\mathcal{T}_b(U, V, W, X)\xi - \mathcal{T}_b(U, V, W, \xi)X \\ & - g(X, U)\mathcal{T}_b(\xi, V)W + \varepsilon\eta(U)\mathcal{T}_b(X, V)W \\ & - g(X, V)\mathcal{T}_b(U, \xi)W + \varepsilon\eta(V)\mathcal{T}_b(U, X)W \\ & - g(X, W)\mathcal{T}_b(U, V)\xi + \varepsilon\eta(W)\mathcal{T}_b(U, V)X). \end{aligned} \quad (4.5)$$

Taking the inner product of (4.5) with ξ , we get

$$\begin{aligned} & \mathcal{T}_a(\xi, X, \mathcal{T}_b(U, V)W, \xi) - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V, W, \xi) \\ & - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V, W, \xi) - \mathcal{T}_b(U, V, \mathcal{T}_a(\xi, X)W, \xi) \\ = & L_g(\varepsilon\mathcal{T}_b(U, V, W, X) - \varepsilon\eta(X)\mathcal{T}_b(U, V, W, \xi) \\ & - g(X, U)\mathcal{T}_b(\xi, V, W, \xi) + \varepsilon\eta(U)\mathcal{T}_b(X, V, W, \xi) \\ & - g(X, V)\mathcal{T}_b(U, \xi, W, \xi) + \varepsilon\eta(V)\mathcal{T}_b(U, X, W, \xi) \\ & - g(X, W)\mathcal{T}_b(U, V, \xi, \xi) + \varepsilon\eta(W)\mathcal{T}_b(U, V, X, \xi)). \end{aligned} \quad (4.6)$$

By using (3.29), ..., (3.34) in (4.6), we get (4.3). ■

Theorem 4.3 *Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned} & -\varepsilon\{a_5b_0 + na_5b_1 + a_5b_2 + a_5b_6 + a_5b_3 + a_5b_5\}S^2(V, W) \\ & + \{(nb_1 + b_2 + b_3 + b_5 + b_6 + b_0)(\varepsilon ka_0 + \varepsilon b_7r) \\ & - \varepsilon k(n-1)(2a_5b_6 + a_2b_3 + a_1b_6 + a_1b_3 + a_1b_5 \\ & + a_1b_1 + a_1b_2 + a_2b_2 + a_2b_6 + na_2b_1 + a_1b_0 + a_2b_0)\} \\ & - \varepsilon(n-1)a_1b_7r - \varepsilon na_5b_7r - \varepsilon b_4a_5r - \varepsilon a_1b_4r)S(V, W) \\ & + \{-\varepsilon k(n-1)(nb_1 + b_2 + b_3 + b_5 + b_6 + b_0)(a_7r + ka_0 + k(n-1)a_4) \\ & - \varepsilon k(n-1)r((n-1)b_7a_2 + (n-1)b_7a_4 + a_2b_4 + a_4b_4)\}g(V, W) \\ & + (a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6)\{-k^2(n-1)^2(nb_1 + b_2 + b_3 + b_5 + b_6 + b_0) \\ & - k(n-1)^2b_7r - k(n-1)b_4r\}\eta(V)\eta(W) \\ = & L_g(\varepsilon(b_0 + b_5 + b_6)S(V, W) + \varepsilon(b_4r - k(n-1)(b_0 + nb_4 + b_5 + b_6))g(V, W) \\ & + (b_2 + b_3)(r - kn(n-1))\eta(V)\eta(W)). \end{aligned} \quad (4.7)$$

Theorem 4.4 Let M be an n -dimensional \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& \varepsilon a_0 k R(U, V, W, X) + \varepsilon k a_4 S(X, U) g(V, W) + \varepsilon k a_5 S(X, V) g(U, W) \\
& + \varepsilon k a_6 S(X, W) g(U, V) - \varepsilon k^2 (n-1) a_6 g(X, W) g(U, V) \\
& - \varepsilon k (k a_0 + k(n-1) a_4) g(V, W) g(X, U) \\
& - \varepsilon k (-k a_0 + k(n-1) a_5) g(U, W) g(X, V) \\
& - k^2 (n-1) (a_2 + a_3) g(X, U) \eta(V) \eta(W) \\
& - k^2 (n-1) (a_1 + a_3) g(X, V) \eta(U) \eta(W) \\
& - k^2 (n-1) (a_1 + a_2) g(X, W) \eta(U) \eta(V) \\
& + k(a_2 + a_3) S(X, U) \eta(V) \eta(W) + k(a_1 + a_3) S(X, V) \eta(U) \eta(W) \\
& + k(a_1 + a_2) S(X, W) \eta(U) \eta(V) \\
& = L_g (\varepsilon a_0 R(U, V, W, X) + \varepsilon a_4 S(X, U) g(V, W) + \varepsilon a_5 S(X, V) g(U, W) \\
& + \varepsilon a_6 S(X, W) g(U, V) - \varepsilon k(n-1) a_6 g(U, V) g(X, W) \\
& - k(n-1) (a_2 + a_3) g(X, U) \eta(V) \eta(W) \\
& - k(n-1) (a_1 + a_3) g(X, V) \eta(U) \eta(W) \\
& - k(n-1) (a_1 + a_2) g(X, W) \eta(U) \eta(V) \\
& + (a_2 + a_3) S(X, U) \eta(V) \eta(W) \\
& + (a_1 + a_3) S(X, V) \eta(U) \eta(W) \\
& + (a_1 + a_2) S(X, W) \eta(U) \eta(V) \\
& - \varepsilon (-k a_0 + k(n-1) a_5) g(U, W) g(X, V) \\
& - \varepsilon (k a_0 + k(n-1) a_4) g(V, W) g(X, U)). \tag{4.8}
\end{aligned}$$

Remark 4.5 Here two cases arise. First is that when $L_g = 0$. In this case, it is \mathcal{T}_a -semisymmetric. We exclude this case, because it is already studied in [28]. And the second case is that when $L_g \neq 0$. In the following Theorem, we consider the result for $L_g \neq 0$.

Theorem 4.6 Let M be an n -dimensional \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that $a_0 + a_5 + a_6 \neq 0$.

1. If $a_0 + a_2 + a_3 + n a_4 + a_5 + a_6 \neq 0$, then either it is Einstein manifold or $L_g = k$.
2. If $a_0 + a_2 + a_3 + n a_4 + a_5 + a_6 = 0$, then either it is η -Einstein manifold or $L_g = k$.

Proof. Let M be an n -dimensional \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. On contracting (4.8), we get

$$\begin{aligned}
& k(\varepsilon(a_0 + a_5 + a_6) S(V, W) + \varepsilon(a_4 r - k(n-1)(a_0 + n a_4 + a_5 + a_6)) g(V, W) \\
& + (a_2 + a_3)(r - k n(n-1)) \eta(V) \eta(W)) \\
& = L_g (\varepsilon(a_0 + a_5 + a_6) S(V, W) + \varepsilon(a_4 r - k(n-1)(a_0 + n a_4 + a_5 + a_6)) g(V, W) \\
& + (a_2 + a_3)(r - k n(n-1)) \eta(V) \eta(W)),
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
& (L_g - k)(\varepsilon(a_0 + a_5 + a_6) S(V, W) \\
& + \varepsilon(a_4 r - k(n-1)(a_0 + n a_4 + a_5 + a_6)) g(V, W) \\
& + (a_2 + a_3)(r - k n(n-1)) \eta(V) \eta(W)). \tag{4.9}
\end{aligned}$$

On contracting above equation, we get

$$(L_g - k)(a_0 + a_2 + a_3 + n a_4 + a_5 + a_6)(r - k n(n-1)).$$

Case 1. If $a_0 + a_2 + a_3 + n a_4 + a_5 + a_6 \neq 0$, then either $L_g = k$ or $r = k n(n-1)$. If $r = k n(n-1)$, then from (4.9), we get

$$S = k(n-1)g.$$

Case 2. If $a_0 + a_2 + a_3 + na_4 + a_5 + a_6 = 0$, then by (4.9), we get either $L_g = k$ or

$$\begin{aligned} -\varepsilon(a_0 + a_5 + a_6)S(V, W) &= \varepsilon(a_4r - k(n-1)(a_0 + na_4 + a_5 + a_6))g(V, W) \\ &\quad + (a_2 + a_3)(r - kn(n-1))\eta(V)\eta(W). \end{aligned}$$

This proves the result. ■

Corollary 4.7 *An n -dimensional \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold is of the form $R \cdot \mathcal{T}_a = kQ(g, \mathcal{T}_a)$.*

In view of Theorem 4.6, we have the following

Corollary 4.8 *Let M be an n -dimensional \mathcal{T}_a -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that*

$$\mathcal{T}_a \in \{R, \mathcal{V}, \mathcal{P}, \mathcal{M}, \mathcal{W}_0, \mathcal{W}_0^*, \mathcal{W}_1, \mathcal{W}_1^*, \mathcal{W}_3, \dots, \mathcal{W}_8\}$$

Then we have the following table:

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 4.9 [16] *Let M be an n -dimensional, $n \geq 3$, Kenmotsu manifold. If M is pseudosymmetric then either it is locally isometric to the hyperbolic space $H^n(-1)$ or $L_g = -1$ holds on M .*

Corollary 4.10 [16] *Every Kenmotsu manifold M , $n \geq 3$, is a pseudosymmetric manifold of the form $R \cdot R = -Q(g, R)$.*

Corollary 4.11 *Let M be an n -dimensional quasi-conformal pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that $a_0 - a_1 \neq 0$ and $a_0 + (n-2)a_1 \neq 0$. Then we have the following table:*

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 4.12 *Let M be an n -dimensional pseudo-projective pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that $a_0 \neq 0$ and $a_0 - a_1 \neq 0$. Then we have the following table:*

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 4.13 *Let M be an n -dimensional Weyl pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$S = \left(\frac{r}{n-1} - k \right) g + \left(nk - \frac{r}{n-1} \right) \eta \otimes \eta$
Sasakian	1	$S = \left(\frac{r}{n-1} - 1 \right) g + \left(n - \frac{r}{n-1} \right) \eta \otimes \eta$
Kenmotsu [16]	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -Sasakian	ε	$S = \left(\frac{r}{n-1} - \varepsilon \right) g + \varepsilon \left(n\varepsilon - \frac{r}{n-1} \right) \eta \otimes \eta$
para-Sasakian [15]	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -para-Sasakian	$-\varepsilon$	$S = \left(\frac{r}{n-1} + \varepsilon \right) g - \varepsilon \left(n\varepsilon + \frac{r}{n-1} \right) \eta \otimes \eta$

Corollary 4.14 *Let M be an n -dimensional conharmonic pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$S = \left(\frac{r}{n-1} - k \right) g + \left(nk - \frac{r}{n-1} \right) \eta \otimes \eta$
Sasakian	1	$S = \left(\frac{r}{n-1} - 1 \right) g + \left(n - \frac{r}{n-1} \right) \eta \otimes \eta$
Kenmotsu	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -Sasakian	ε	$S = \left(\frac{r}{n-1} - \varepsilon \right) g + \varepsilon \left(n\varepsilon - \frac{r}{n-1} \right) \eta \otimes \eta$
para-Sasakian	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -para-Sasakian	$-\varepsilon$	$S = \left(\frac{r}{n-1} + \varepsilon \right) g - \varepsilon \left(n\varepsilon + \frac{r}{n-1} \right) \eta \otimes \eta$

Corollary 4.15 *Let M be an n -dimensional \mathcal{W}_2 -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$\frac{r}{n}g$
Sasakian	1	$\frac{r}{n}g$
Kenmotsu	-1	$\frac{r}{n}g$
(ε) -Sasakian	ε	$\frac{r}{n}g$
para-Sasakian	-1	$\frac{r}{n}g$
(ε) -para-Sasakian	$-\varepsilon$	$\frac{r}{n}g$

Corollary 4.16 *Let M be an n -dimensional \mathcal{W}_9 -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.*

Then we have the following table:

M	$L_g =$	$S =$
$N(k)$ -contact metric	k	$S = \left(\frac{r}{n-1} - k \right) g + \left(nk - \frac{r}{n-1} \right) \eta \otimes \eta$
Sasakian	1	$S = \left(\frac{r}{n-1} - 1 \right) g + \left(n - \frac{r}{n-1} \right) \eta \otimes \eta$
Kenmotsu	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -Sasakian	ε	$S = \left(\frac{r}{n-1} - \varepsilon \right) g + \varepsilon \left(n\varepsilon - \frac{r}{n-1} \right) \eta \otimes \eta$
para-Sasakian	-1	$S = \left(\frac{r}{n-1} + 1 \right) g - \left(n + \frac{r}{n-1} \right) \eta \otimes \eta$
(ε) -para-Sasakian	$-\varepsilon$	$S = \left(\frac{r}{n-1} + \varepsilon \right) g - \varepsilon \left(n\varepsilon + \frac{r}{n-1} \right) \eta \otimes \eta$

5 $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetry

Definition 5.1 A semi-Riemannian manifold is said to be $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric if it satisfies

$$\mathcal{T}_a \cdot \mathcal{T}_b = L_{S^\ell} Q(S^\ell, \mathcal{T}_b), \quad (5.1)$$

where L_{S^ℓ} is some smooth function on M . In particular, it is said to be $(R, \mathcal{T}_a, S^\ell)$ -pseudosymmetric if

$$R \cdot \mathcal{T}_a = L_{S^\ell} Q(S^\ell, \mathcal{T}_a) \quad (5.2)$$

holds on the set $\mathcal{U}_{S^\ell} = \{x \in M : Q(S^\ell, \mathcal{T}_a) \neq 0\}$, where L_{S^ℓ} is some smooth function on \mathcal{U}_{S^ℓ} .

For $\ell = 1$, we can give the following definition.

Definition 5.2 A semi-Riemannian manifold is called $(\mathcal{T}_a, \mathcal{T}_b, S)$ -pseudosymmetric if it satisfies

$$\mathcal{T}_a \cdot \mathcal{T}_b = L_S Q(S, \mathcal{T}_b), \quad (5.3)$$

where L_S is some smooth function on M . In particular, it is said to be (R, \mathcal{T}_a, S) -pseudosymmetric if

$$R \cdot \mathcal{T}_a = L_S Q(S, \mathcal{T}_a) \quad (5.4)$$

holds on the set $\mathcal{U}_S = \{x \in M : Q(S, \mathcal{T}_a) \neq 0\}$, where L_S is some smooth function on \mathcal{U}_S .

Remark 5.3 A semi-Riemannian manifold is said to be (R, R, S) -pseudosymmetric or in short, Ricci-generalized pseudosymmetric if

$$R \cdot R = L_S Q(S, R)$$

holds on the set $\mathcal{U}_S = \{x \in M : Q(S, R) \neq 0\}$, where L_S is some smooth function on \mathcal{U}_S . It is known [5] that every 3-dimensional semi-Riemannian manifold is Ricci-generalized pseudosymmetric along with $L_S = 1$, that is, $R \cdot R = Q(S, R)$.

Theorem 5.4 Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian mani-

fold. Then

$$\begin{aligned}
& \varepsilon b_0(ka_0 + \varepsilon k(n-1)a_4 + a_7r)R(U, V, W, X) + \varepsilon a_1b_0S(X, R(U, V)W) \\
& - 2k(n-1)a_3(kb_0 + k(n-1)b_4 + b_7r)\eta(X)\eta(U)g(V, W) \\
& - 2k(n-1)a_3(-kb_0 + k(n-1)b_5 - b_7r)\eta(X)\eta(V)g(U, W) \\
& + \varepsilon a_1b_4S^2(X, U)g(V, W) + \varepsilon a_1b_5S^2(X, V)g(U, W) \\
& + \varepsilon a_1b_6S^2(X, W)g(U, V) - a_5(b_1 + b_3)S^2(X, V)\eta(U)\eta(W) \\
& - a_5(b_1 + b_2)S^2(X, W)\eta(U)\eta(V) - a_5(b_2 + b_3)S^2(X, U)\eta(V)\eta(W) \\
& - 2a_6b_1S^2(V, W)\eta(X)\eta(U) - 2a_6b_2S^2(U, W)\eta(X)\eta(V) \\
& - 2a_6b_3S^2(U, V)\eta(X)\eta(W) - 2k^2(n-1)a_3b_6g(U, V)\eta(X)\eta(W) \\
& - 2(k(n-1)a_3b_1 + a_6(kb_0 + k(n-1)b_4 + b_7r))\eta(X)\eta(U)S(V, W) \\
& - 2(k(n-1)a_3b_2 + a_6(-kb_0 + k(n-1)b_5 - b_7r))\eta(X)\eta(V)S(U, W) \\
& - 2k(n-1)(a_3b_3 + a_6b_6)S(U, V)\eta(X)\eta(W) \\
& + \varepsilon(b_4(ka_0 + k(n-1)a_4 + a_7r) - a_1(kb_0 + k(n-1)b_4))S(X, U)g(V, W) \\
& + \varepsilon(b_5(ka_0 + k(n-1)a_4 + a_7r) - a_1(-kb_0 + k(n-1)b_5))S(X, V)g(U, W) \\
& + \varepsilon b_6(ka_0 + k(n-1)(a_4 - a_1) + a_7r)S(X, W)g(U, V) \\
& - \varepsilon(kb_0 + k(n-1)b_4)(ka_0 + k(n-1)a_4 + a_7r)g(X, U)g(V, W) \\
& - \varepsilon(-kb_0 + k(n-1)b_5)(ka_0 + k(n-1)a_4 + a_7r)g(U, W)g(X, V) \\
& - \varepsilon k(n-1)b_6(ka_0 + k(n-1)a_4 + a_7r)g(X, W)g(U, V) \\
& - k(n-1)((b_2 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(-kb_0 + k(n-1)(b_5 + b_6) - b_7r))g(X, U)\eta(V)\eta(W) \\
& - k(n-1)((b_1 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))g(X, V)\eta(U)\eta(W) \\
& - ((b_1 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))S(X, V)\eta(U)\eta(W) \\
& - ((b_2 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_5 + b_6) + b_7r))S(X, U)\eta(V)\eta(W) \\
& - ((b_1 + b_2)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + k(n-1)(b_4 + b_5)(a_1 + a_5))S(X, W)\eta(U)\eta(V) \\
& - k(n-1)(k(n-1)(b_4 + b_5)(a_2 + a_4) \\
& + (b_1 + b_2)(ka_0 + k(n-1)a_4 + a_7r))g(X, W)\eta(U)\eta(V) \\
= & L_{S^\ell}(\varepsilon b_0S^\ell(R(U, V)W, X) + \varepsilon b_4S^{\ell+1}(X, U)g(V, W) + \varepsilon b_5S^{\ell+1}(X, V)g(U, W) \\
& + \varepsilon b_6S^{\ell+1}(X, W)g(U, V) - \varepsilon k(n-1)b_6S^\ell(X, W)g(U, V) \\
& + k^\ell(n-1)^\ell(-kb_0 + k(n-1)(b_5 + b_6) - b_7r)g(X, U)\eta(V)\eta(W) \\
& + k^\ell(n-1)^\ell(kb_0 + k(n-1)(b_4 + b_6) + b_7r)g(X, V)\eta(U)\eta(W) \\
& - (-kb_0 + k(n-1)(b_2 + b_3 + b_5 + b_6) - b_7r)S^\ell(X, U)\eta(V)\eta(W) \\
& - (kb_0 + k(n-1)(b_1 + b_3 + b_4 + b_6) + b_7r)S^\ell(X, V)\eta(U)\eta(W) \\
& - k(n-1)(b_1 + b_2 + b_4 + b_5)S(X, W)\eta(U)\eta(V) \\
& + k^{\ell+1}(n-1)^{\ell+1}(b_4 + b_5)g(X, W)\eta(U)\eta(V) \\
& - \varepsilon(-kb_0 + k(n-1)b_5)S^\ell(X, V)g(U, W) \\
& - \varepsilon(kb_0 + k(n-1)b_4)S^\ell(X, U)g(V, W) \\
& + k^\ell(n-1)^\ell(b_1 + b_3)S(X, V)\eta(U)\eta(W) \\
& + k^\ell(n-1)^\ell(b_1 + b_2)S(X, W)\eta(U)\eta(V) \\
& + k^\ell(n-1)^\ell(b_2 + b_3)S(X, U)\eta(V)\eta(W)).
\end{aligned} \tag{5.5}$$

Proof. Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

Then

$$\mathcal{T}_a(Z, X) \cdot \mathcal{T}_b(U, V)W = L_{S^\ell}Q(S^\ell, \mathcal{T}_b)(U, V, W; Z, X). \quad (5.6)$$

Taking $Z = \xi$ in (5.6), we get

$$\mathcal{T}_a(\xi, X) \cdot \mathcal{T}_b(U, V)W = L_{S^\ell}Q(S^\ell, \mathcal{T}_b)(U, V, W; \xi, X),$$

which gives

$$\begin{aligned} & \mathcal{T}_a(\xi, X)\mathcal{T}_b(U, V)W - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V)W \\ & \quad - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V)W - \mathcal{T}_b(U, V)\mathcal{T}_a(\xi, X)W \\ = & L_{S^\ell}((\xi \wedge_{S^\ell} X)\mathcal{T}_b(U, V)W - \mathcal{T}_b((\xi \wedge_{S^\ell} X)U, V)W \\ & \quad - \mathcal{T}_b(U, (\xi \wedge_{S^\ell} X)V)W - \mathcal{T}_b(U, V)(\xi \wedge_{S^\ell} X)W), \end{aligned}$$

that is,

$$\begin{aligned} & \mathcal{T}_a(\xi, X)\mathcal{T}_b(U, V)W - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V)W \\ & \quad - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V)W - \mathcal{T}_b(U, V)\mathcal{T}_a(\xi, X)W \\ = & L_{S^\ell}(S^\ell(X, \mathcal{T}_b(U, V)W)\xi - S^\ell(\xi, \mathcal{T}_b(U, V)W)X \\ & \quad - S^\ell(X, U)\mathcal{T}_b(\xi, V)W + S^\ell(\xi, U)\mathcal{T}_b(X, V)W \\ & \quad - S^\ell(X, V)\mathcal{T}_b(U, \xi)W + S^\ell(\xi, V)\mathcal{T}_b(U, X)W \\ & \quad - S^\ell(X, W)\mathcal{T}_b(U, V)\xi + S^\ell(\xi, W)\mathcal{T}_b(U, V)X). \end{aligned} \quad (5.7)$$

Taking the inner product of (5.7) with ξ , we get

$$\begin{aligned} & \mathcal{T}_a(\xi, X, \mathcal{T}_b(U, V)W, \xi) - \mathcal{T}_b(\mathcal{T}_a(\xi, X)U, V, W, \xi) \\ & \quad - \mathcal{T}_b(U, \mathcal{T}_a(\xi, X)V, W, \xi) - \mathcal{T}_b(U, V, \mathcal{T}_a(\xi, X)W, \xi) \\ = & L_{S^\ell}(\varepsilon S^\ell(X, \mathcal{T}_b(U, V)W) - \varepsilon \eta(X)S^\ell(\xi, \mathcal{T}_b(U, V)W) \\ & \quad - S^\ell(X, U)\mathcal{T}_b(\xi, V, W, \xi) + S^\ell(\xi, U)\mathcal{T}_b(X, V, W, \xi) \\ & \quad - S^\ell(X, V)\mathcal{T}_b(U, \xi, W, \xi) + S^\ell(\xi, V)\mathcal{T}_b(U, X, W, \xi) \\ & \quad - S^\ell(X, W)\mathcal{T}_b(U, V, \xi, \xi) + S^\ell(\xi, W)\mathcal{T}_b(U, V, X, \xi)). \end{aligned} \quad (5.8)$$

By using (3.29), ..., (3.34) in (5.8), we get (5.5). ■

Corollary 5.5 *Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b, S)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian mani-*

fold. Then

$$\begin{aligned}
& \varepsilon b_0(ka_0 + \varepsilon k(n-1)a_4 + a_7r)R(U, V, W, X) + \varepsilon a_1 b_0 S(X, R(U, V)W) \\
& - 2k(n-1)a_3(kb_0 + k(n-1)b_4 + b_7r)\eta(X)\eta(U)g(V, W) \\
& - 2k(n-1)a_3(-kb_0 + k(n-1)b_5 - b_7r)\eta(X)\eta(V)g(U, W) \\
& + \varepsilon a_1 b_4 S^2(X, U)g(V, W) + \varepsilon a_1 b_5 S^2(X, V)g(U, W) \\
& + \varepsilon a_1 b_6 S^2(X, W)g(U, V) - a_5(b_1 + b_3)S^2(X, V)\eta(U)\eta(W) \\
& - a_5(b_1 + b_2)S^2(X, W)\eta(U)\eta(V) - a_5(b_2 + b_3)S^2(X, U)\eta(V)\eta(W) \\
& - 2a_6 b_1 S^2(V, W)\eta(X)\eta(U) - 2a_6 b_2 S^2(U, W)\eta(X)\eta(V) \\
& - 2a_6 b_3 S^2(U, V)\eta(X)\eta(W) - 2k^2(n-1)a_3 b_6 g(U, V)\eta(X)\eta(W) \\
& - 2(k(n-1)a_3 b_1 + a_6(kb_0 + k(n-1)b_4 + b_7r))\eta(X)\eta(U)S(V, W) \\
& - 2(k(n-1)a_3 b_2 + a_6(-kb_0 + k(n-1)b_5 - b_7r))\eta(X)\eta(V)S(U, W) \\
& - 2k(n-1)(a_3 b_3 + a_6 b_6)S(U, V)\eta(X)\eta(W) \\
& + \varepsilon(b_4(ka_0 + k(n-1)a_4 + a_7r) - a_1(kb_0 + k(n-1)b_4))S(X, U)g(V, W) \\
& + \varepsilon(b_5(ka_0 + k(n-1)a_4 + a_7r) - a_1(-kb_0 + k(n-1)b_5))S(X, V)g(U, W) \\
& + \varepsilon b_6(ka_0 + k(n-1)(a_4 - a_1) + a_7r)S(X, W)g(U, V) \\
& - \varepsilon(kb_0 + k(n-1)b_4)(ka_0 + k(n-1)a_4 + a_7r)g(X, U)g(V, W) \\
& - \varepsilon(-kb_0 + k(n-1)b_5)(ka_0 + k(n-1)a_4 + a_7r)g(U, W)g(X, V) \\
& - \varepsilon k(n-1)b_6(ka_0 + k(n-1)a_4 + a_7r)g(X, W)g(U, V) \\
& - k(n-1)((b_2 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(-kb_0 + k(n-1)(b_5 + b_6) - b_7r))g(X, U)\eta(V)\eta(W) \\
& - k(n-1)((b_1 + b_3)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))g(X, V)\eta(U)\eta(W) \\
& - ((b_1 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))S(X, V)\eta(U)\eta(W) \\
& - ((b_2 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + (a_1 + a_5)(kb_0 + k(n-1)(b_5 + b_6) + b_7r))S(X, U)\eta(V)\eta(W) \\
& - ((b_1 + b_2)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& + k(n-1)(b_4 + b_5)(a_1 + a_5))S(X, W)\eta(U)\eta(V) \\
& - k(n-1)(k(n-1)(b_4 + b_5)(a_2 + a_4) \\
& + (b_1 + b_2)(ka_0 + k(n-1)a_4 + a_7r))g(X, W)\eta(U)\eta(V) \\
= & L_S(\varepsilon b_0 S(R(U, V)W, X) + \varepsilon b_4 S^2(X, U)g(V, W) + \varepsilon b_5 S^2(X, V)g(U, W) \\
& + \varepsilon b_6 S^2(X, W)g(U, V) - \varepsilon k(n-1)b_6 S(X, W)g(U, V) \\
& + k(n-1)(-kb_0 + k(n-1)(b_5 + b_6) - b_7r)g(X, U)\eta(V)\eta(W) \\
& + k(n-1)(kb_0 + k(n-1)(b_4 + b_6)b_7r)g(X, V)\eta(U)\eta(W) \\
& + k^2(n-1)^2(b_4 + b_5)g(X, W)\eta(U)\eta(V) \\
& - (-kb_0 + k(n-1)(b_5 + b_6) - b_7r)S(X, U)\eta(V)\eta(W) \\
& - (kb_0 + k(n-1)(b_4 + b_6) + b_7r)S(X, V)\eta(U)\eta(W) \\
& - k(n-1)(b_4 + b_5)S(X, W)\eta(U)\eta(V) - \varepsilon(-kb_0 + k(n-1)b_5)S(X, V)g(U, W) \\
& - \varepsilon(kb_0 + k(n-1)b_4)S(X, U)g(V, W)). \tag{5.9}
\end{aligned}$$

Theorem 5.6 Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& (a_1b_5 - a_5b_1 - a_5b_3)S^2(X, V)\eta(W) \\
& + (a_1b_6 - a_5b_1 - a_5b_2)S^2(X, W)\eta(V) \\
& - 2a_6b_1S^2(V, W)\eta(X) \\
& + (b_5(ka_0 + k(n-1)(a_4 - a_1) + a_7r) \\
& - (b_1 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& - (a_1 + a_5)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))S(X, V)\eta(W) \\
& + (b_6(ka_0 + k(n-1)(a_4 - a_1) + a_7r) \\
& - (b_1 + b_2)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& - k(n-1)(a_1 + a_5)(b_4 + b_5))S(X, W)\eta(V) \\
& - 2(k(n-1)a_3b_1 + a_6(kb_0 + k(n-1)b_4 + b_7r))S(V, W)\eta(X) \\
& - k(n-1)((b_1 + b_3 + b_5)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))g(X, V)\eta(W) \\
& - k(n-1)((b_1 + b_2 + b_6)(ka_0 + k(n-1)a_4 + a_7r) \\
& + k(n-1)(a_2 + a_4)(b_4 + b_5))g(X, W)\eta(V) \\
& - 2k(n-1)a_3(kb_0 + k(n-1)b_4 + b_7r)g(V, W)\eta(X) \\
& - \varepsilon k(n-1)((kb_0 + b_7r)(a_1 + a_2 + a_4 + a_5) \\
& + k(n-1)(b_2 + b_3 + b_5 + b_6) \times \\
& (a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6))\eta(X)\eta(V)\eta(W) \\
& = L_{S^\ell}(-(kb_0 + k(n-1)(b_1 + b_3 + b_4 + b_5 + b_6) + b_7r)S^\ell(X, V)\eta(W) \\
& + k^\ell(n-1)^\ell(kb_0 + k(n-1)(b_4 + b_6) + b_7r)g(X, V)\eta(W) \\
& + k^{\ell+1}(n-1)^{\ell+1}(b_4 + b_5)g(X, W)\eta(V) \\
& - k(n-1)(b_1 + b_2 + b_4 + b_5 + b_6)S^\ell(X, W)\eta(V) \\
& + b_5S^{\ell+1}(X, V)\eta(W) + b_6S^{\ell+1}(X, W)\eta(V)) \\
& + k^\ell(n-1)^\ell(b_1 + b_3)S(X, V)\eta(W) \\
& + k^\ell(n-1)^\ell(b_1 + b_2)S(X, W)\eta(V)).
\end{aligned}$$

Corollary 5.7 *Let M be an n -dimensional $(\mathcal{T}_a, \mathcal{T}_b, S)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& (a_1b_5 - a_5b_1 - a_5b_3)S^2(X, V)\eta(W) \\
& + (a_1b_6 - a_5b_1 - a_5b_2)S^2(X, W)\eta(V) - 2a_6b_1S^2(V, W)\eta(X) \\
& + (b_5(ka_0 + k(n-1)(a_4 - a_1) + a_7r) \\
& - (b_1 + b_3)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& - (a_1 + a_5)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))S(X, V)\eta(W) \\
& + (b_6(ka_0 + k(n-1)(a_4 - a_1) + a_7r) \\
& - (b_1 + b_2)(-ka_0 + k(n-1)(a_1 + a_2) - a_7r) \\
& - k(n-1)(a_1 + a_5)(b_4 + b_5))S(X, W)\eta(V) \\
& - 2(k(n-1)a_3b_1 + a_6(kb_0 + k(n-1)b_4 + b_7r))S(V, W)\eta(X) \\
& - k(n-1)((b_1 + b_3 + b_5)(ka_0 + k(n-1)a_4 + a_7r) \\
& + (a_2 + a_4)(kb_0 + k(n-1)(b_4 + b_6) + b_7r))g(X, V)\eta(W) \\
& - k(n-1)((b_1 + b_2 + b_6)(ka_0 + k(n-1)a_4 + a_7r) \\
& + k(n-1)(a_2 + a_4)(b_4 + b_5))g(X, W)\eta(V) \\
& - 2k(n-1)a_3(kb_0 + k(n-1)b_4 + b_7r)g(V, W)\eta(X) \\
& - \varepsilon k(n-1)((kb_0 + b_7r)(a_1 + a_2 + a_4 + a_5) \\
& + k(n-1)(b_2 + b_3 + b_5 + b_6) \times \\
& (a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6))\eta(X)\eta(V)\eta(W)
\end{aligned}$$

$$\begin{aligned}
&= L_S(-(kb_0 + k(n-1)(b_4 + b_5 + b_6) + b_7r)S(X, V)\eta(W) \\
&\quad + k(n-1)(kb_0 + k(n-1)(b_4 + b_6) + b_7r)g(X, V)\eta(W) \\
&\quad + k^2(n-1)^2(b_4 + b_5)g(X, W)\eta(V) \\
&\quad - k(n-1)(b_4 + b_5 + b_6)S(X, W)\eta(V) \\
&\quad + b_5S^2(X, V)\eta(W) + b_6S^2(X, W)\eta(V)).
\end{aligned} \tag{5.10}$$

Theorem 5.8 *Let M be an n -dimensional $(R, \mathcal{T}_a, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
&k(a_1 + a_3 + a_5)(S(X, V) - k(n-1)g(X, V))\eta(W) \\
&+ k(a_1 + a_2 + a_6)(S(X, W) - k(n-1)g(X, W))\eta(V) \\
&= L_{S^\ell}(-(ka_0 + k(n-1)(a_1 + a_3 + a_4 + a_5 + a_6) + a_7r)S^\ell(X, V)\eta(W) \\
&\quad + k^\ell(n-1)^\ell(ka_0 + k(n-1)(a_4 + a_6) + a_7r)g(X, V)\eta(W) \\
&\quad + k^{\ell+1}(n-1)^{\ell+1}(a_4 + a_5)g(X, W)\eta(V) \\
&\quad - k(n-1)(a_1 + a_2 + a_4 + a_5 + a_6)S^\ell(X, W)\eta(V) \\
&\quad + a_5S^{\ell+1}(X, V)\eta(W) + a_6S^{\ell+1}(X, W)\eta(V) \\
&\quad + k^\ell(n-1)^\ell(a_1 + a_3)S(X, V)\eta(W) \\
&\quad + k^\ell(n-1)^\ell(a_1 + a_2)S(X, W)\eta(V)).
\end{aligned} \tag{5.11}$$

Corollary 5.9 *Let M be an n -dimensional (R, \mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
&k(a_1 + a_3 + a_5)(S(X, V) - k(n-1)g(X, V))\eta(W) \\
&+ k(a_1 + a_2 + a_6)(S(X, W) - k(n-1)g(X, W))\eta(V) \\
&= L_S(-(ka_0 + k(n-1)(a_4 + a_5 + a_6) + a_7r)S(X, V)\eta(W) \\
&\quad + k(n-1)(ka_0 + k(n-1)(a_4 + a_6) + a_7r)g(X, V)\eta(W) \\
&\quad + k^2(n-1)^2(a_4 + a_5)g(X, W)\eta(V) \\
&\quad - k(n-1)(a_4 + a_5 + a_6)S(X, W)\eta(V) \\
&\quad + a_5S^2(X, V)\eta(W) + a_6S^2(X, W)\eta(V)).
\end{aligned} \tag{5.12}$$

Corollary 5.10 *Let M be an n -dimensional (R, R, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not semisymmetric, then*

$$S^\ell = k^\ell(n-1)^\ell g$$

and $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	$S^\ell =$
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$k^\ell(n-1)^\ell g$
Sasakian	$\frac{1}{(n-1)^\ell}$	$(n-1)^\ell g$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -Sasakian	$\frac{1}{(\varepsilon)^{\ell-1}(n-1)^\ell}$	$(\varepsilon)^\ell(n-1)^\ell g$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$(-\varepsilon)^\ell(n-1)^\ell g$

Proof. Let M be an n -dimensional (R, R, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold, that is

$$R \cdot R = L_{S^\ell}Q(S^\ell, R) \tag{5.13}$$

holds on M . By putting the value for R in (5.11), we get

$$-kL_{S^\ell} (S^\ell(X, V) - k^\ell(n-1)^\ell g(X, V)) \eta(W) = 0. \quad (5.14)$$

Putting $W = \xi$ in (5.14), we get

$$-kL_{S^\ell} (S^\ell(X, V) - k^\ell(n-1)^\ell g(X, V)) = 0. \quad (5.15)$$

Since M is not semisymmetric $L_{S^\ell} \neq 0$. Therefore from (5.15), we have

$$S^\ell(X, V) = k^\ell(n-1)^\ell g(X, V).$$

So putting $S^\ell = k^\ell(n-1)^\ell g$ in (5.13), we get

$$R \cdot R = k^\ell(n-1)^\ell L_{S^\ell} Q(g, R),$$

which is the condition of pseudosymmetric manifold. By comparison with the result of pseudosymmetric manifold (Corollary 4.8), we get $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. This proves the result. ■

Corollary 5.11 *Let M be an n -dimensional Ricci-generalized pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:*

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu [16]	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.12 *Let M be an n -dimensional (R, C_*, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not quasi-conformal semisymmetric, then*

$$\begin{aligned} S^{\ell+1} &= \left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - k(n-1) \right) \right) (S^\ell - k^\ell(n-1)^\ell g) \\ &\quad + k^\ell(n-1)^\ell S. \end{aligned}$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$\left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - k(n-1) \right) \right) (S^\ell - k^\ell(n-1)^\ell g) + k^\ell(n-1)^\ell S$
Sasakian	$\left(\left(\frac{r}{n(n-1)} - 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - 1(n-1) \right) \right) (S^\ell - (n-1)^\ell g) + (n-1)^\ell S$
Kenmotsu	$\left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + (n-1) \right) \right) (S^\ell - (-1)^\ell(n-1)^\ell g) + (-1)^\ell(n-1)^\ell S$
(ε) -Sasakian	$\left(\left(\frac{r}{n(n-1)} - \varepsilon \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - \varepsilon(n-1) \right) \right) (S^\ell - (\varepsilon)^\ell(n-1)^\ell g) + (\varepsilon)^\ell(n-1)^\ell S$
para-Sasakian	$\left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + (n-1) \right) \right) (S^\ell - (-1)^\ell(n-1)^\ell g) + (-1)^\ell(n-1)^\ell S$
(ε) -para-Sasakian	$\left(\left(\frac{r}{n(n-1)} + \varepsilon \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + \varepsilon(n-1) \right) \right) (S^\ell - (-\varepsilon)^\ell(n-1)^\ell g) + (-\varepsilon)^\ell(n-1)^\ell S$

Corollary 5.13 *Let M be an n -dimensional (R, \mathcal{C}_*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not quasi-conformal-semisymmetric, then*

$$S^2 = \left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S - k(n-1) \left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - k(n-1) \right) \right) g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$\left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S - k(n-1) \left(\left(\frac{r}{n(n-1)} - k \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - k(n-1) \right) \right) g$
Sasakian	$\left(\left(\frac{r}{n(n-1)} - 1 \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S - (n-1) \left(\left(\frac{r}{n(n-1)} - 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - (n-1) \right) \right) g$
Kenmotsu	$\left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S + (n-1) \left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + (n-1) \right) \right) g$
(ε) -Sasakian	$\left(\left(\frac{r}{n(n-1)} - \varepsilon \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S - \varepsilon(n-1) \left(\left(\frac{r}{n(n-1)} - \varepsilon \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} - \varepsilon(n-1) \right) \right) g$
para-Sasakian	$\left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S + (n-1) \left(\left(\frac{r}{n(n-1)} + 1 \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + (n-1) \right) \right) g$
(ε) -para-Sasakian	$\left(\left(\frac{r}{n(n-1)} + \varepsilon \right) \frac{a_0}{a_1} + \frac{2r}{n} \right) S + \varepsilon(n-1) \left(\left(\frac{r}{n(n-1)} + \varepsilon \right) \frac{a_0}{a_1} + \left(\frac{2r}{n} + \varepsilon(n-1) \right) \right) g$

Corollary 5.14 *Let M be an n -dimensional (R, \mathcal{C}, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not Weyl-semisymmetric, then*

$$S^{\ell+1} = \left(\frac{r}{n-1} - k \right) (S^\ell - k^\ell(n-1)^\ell g) + k^\ell(n-1)^\ell S.$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$\left(\frac{r}{n-1} - k \right) (S^\ell - k^\ell(n-1)^\ell g) + k^\ell(n-1)^\ell S$
Sasakian	$\left(\frac{r}{n-1} - 1 \right) (S^\ell - (n-1)^\ell g) + (n-1)^\ell S$
Kenmotsu	$\left(\frac{r}{n-1} + 1 \right) (S^\ell - (-1)^\ell(n-1)^\ell g) + (-1)^\ell(n-1)^\ell S$
(ε) -Sasakian	$\left(\frac{r}{n-1} - \varepsilon \right) (S^\ell - (\varepsilon)^\ell(n-1)^\ell g) + (\varepsilon)^\ell(n-1)^\ell S$
para-Sasakian	$\left(\frac{r}{n-1} + 1 \right) (S^\ell - (-1)^\ell(n-1)^\ell g) + (-1)^\ell(n-1)^\ell S$
(ε) -para-Sasakian	$\left(\frac{r}{n-1} + \varepsilon \right) (S^\ell - (-\varepsilon)^\ell(n-1)^\ell g) + (-\varepsilon)^\ell(n-1)^\ell S$

Corollary 5.15 *Let M be an n -dimensional (R, \mathcal{C}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not Weyl-semisymmetric, then*

$$S^2 = \left(k(n-2) + \frac{r}{n-1} \right) S + k(n-1) \left(k - \frac{r}{n-1} \right) g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$\left(k(n-2) + \frac{r}{n-1} \right) S + k(n-1) \left(k - \frac{r}{n-1} \right) g$
Sasakian	$\left((n-2) + \frac{r}{n-1} \right) S + (n-1) \left(1 - \frac{r}{n-1} \right) g$
Kenmotsu	$\left(-(n-2) + \frac{r}{n-1} \right) S + (n-1) \left(1 + \frac{r}{n-1} \right) g$
(ε) -Sasakian	$\left(\varepsilon(n-2) + \frac{r}{n-1} \right) S + \varepsilon(n-1) \left(\varepsilon - \frac{r}{n-1} \right) g$
para-Sasakian	$\left(-(n-2) + \frac{r}{n-1} \right) S + (n-1) \left(1 + \frac{r}{n-1} \right) g$
(ε) -para-Sasakian	$\left(-\varepsilon(n-2) + \frac{r}{n-1} \right) S + \varepsilon(n-1) \left(\varepsilon + \frac{r}{n-1} \right) g$

Corollary 5.16 *Let M be an n -dimensional (R, \mathcal{L}, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not conharmonic semisymmetric, then*

$$S^{\ell+1} = -kS^\ell + k^\ell(n-1)^\ell S + k^{\ell+1}(n-1)^\ell g.$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$-kS^\ell + k^\ell(n-1)^\ell S + k^{\ell+1}(n-1)^\ell g$
Sasakian	$-S^\ell + (n-1)^\ell S + (n-1)^\ell g$
Kenmotsu	$S^\ell + (-1)^\ell(n-1)^\ell S + (-1)^{\ell+1}(n-1)^\ell g$
(ε) -Sasakian	$-\varepsilon S^\ell + (\varepsilon)^\ell(n-1)^\ell S + (\varepsilon)^{\ell+1}(n-1)^\ell g$
para-Sasakian	$S^\ell + (-1)^\ell(n-1)^\ell S + (-1)^{\ell+1}(n-1)^\ell g$
(ε) -para-Sasakian	$\varepsilon S^\ell + (-\varepsilon)^\ell(n-1)^\ell S + (-\varepsilon)^{\ell+1}(n-1)^\ell g$

Corollary 5.17 *Let M be an n -dimensional (R, \mathcal{L}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not conharmonic semisymmetric, then*

$$S^2 = k(n-2)S + k^2(n-1)g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$k(n-2)S + k^2(n-1)g$
Sasakian	$(n-2)S + (n-1)g$
Kenmotsu	$-(n-2)S + (n-1)g$
(ε) -Sasakian	$\varepsilon(n-2)S + (n-1)g$
para-Sasakian	$-(n-2)S + (n-1)g$
(ε) -para-Sasakian	$-\varepsilon(n-2)S + (n-1)g$

Corollary 5.18 *Let M be an n -dimensional (R, \mathcal{V}, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not concircular semisymmetric, then M either satisfies*

$$S^\ell = k^\ell(n-1)^\ell g$$

or scalar curvature is $kn(n-1)$ and $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	Result
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$S^\ell = k^\ell(n-1)^\ell g$ or $r = kn(n-1)$
Sasakian	$\frac{1}{(n-1)^\ell}$	$S^\ell = (n-1)^\ell g$ or $r = n(n-1)$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$S^\ell = (-1)^\ell(n-1)^\ell g$ or $r = -n(n-1)$
(ε) -Sasakian	$\frac{1}{(\varepsilon)^{\ell-1}(n-1)^\ell}$	$S^\ell = (\varepsilon)^\ell(n-1)^\ell g$ or $r = \varepsilon n(n-1)$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$S^\ell = (-1)^\ell(n-1)^\ell g$ or $r = -n(n-1)$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$S^\ell = (-\varepsilon)^\ell(n-1)^\ell g$ or $r = -\varepsilon n(n-1)$

Corollary 5.19 *Let M be an n -dimensional (R, \mathcal{V}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not concircularly semisymmetric, then M is either an Einstein manifold or scalar curvature is $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:*

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$S = k(n-1)g$ or $r = kn(n-1)$
Sasakian	$\frac{1}{n-1}$	$S = (n-1)g$ or $r = n(n-1)$
Kenmotsu [16]	$\frac{1}{n-1}$	$S = -(n-1)g$ or $r = -n(n-1)$
(ε) -Sasakian	$\frac{1}{n-1}$	$S = \varepsilon(n-1)g$ or $r = \varepsilon n(n-1)$
para-Sasakian	$\frac{1}{n-1}$	$S = -(n-1)g$ or $r = -n(n-1)$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$S = -\varepsilon(n-1)g$ or $r = -\varepsilon n(n-1)$

Corollary 5.20 *Let M be an n -dimensional $(R, \mathcal{P}_*, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that $a_0 + (n-1)a_1 \neq 0$. If M is not pseudo-projective semisymmetric, then*

$$\begin{aligned} & \left(\left(k - \frac{r}{n(n-1)} \right) a_0 + \left(k(n-1) - \frac{r}{n} \right) a_1 \right) S^\ell \\ = & k^\ell (n-1)^\ell \left(\left(k - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right) g \\ & + k^\ell (n-1)^\ell a_1 S. \end{aligned}$$

Consequently, we have the following:

M	$S^\ell =$
$N(k)$ -contact metric	$\frac{k^\ell (n-1)^\ell a_1}{\left(\left(k - \frac{r}{n(n-1)} \right) a_0 + \left(k(n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{k^\ell (n-1)^\ell \left(\left(k - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(k - \frac{r}{n(n-1)} \right) a_0 + \left(k(n-1) - \frac{r}{n} \right) a_1 \right)} g$
Sasakian	$\frac{(n-1)^\ell a_1}{\left(\left(1 - \frac{r}{n(n-1)} \right) a_0 + \left((n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{(n-1)^\ell \left(\left(1 - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(1 - \frac{r}{n(n-1)} \right) a_0 + \left((n-1) - \frac{r}{n} \right) a_1 \right)} g$
Kenmotsu	$\frac{(-1)^\ell (n-1)^\ell a_1}{\left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 + \left(-(n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{(-1)^\ell (n-1)^\ell \left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 + \left(-(n-1) - \frac{r}{n} \right) a_1 \right)} g$
(ε) -Sasakian	$\frac{(\varepsilon)^\ell (n-1)^\ell a_1}{\left(\left(\varepsilon - \frac{r}{n(n-1)} \right) a_0 + \left(\varepsilon(n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{(\varepsilon)^\ell (n-1)^\ell \left(\left(\varepsilon - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(\varepsilon - \frac{r}{n(n-1)} \right) a_0 + \left(\varepsilon(n-1) - \frac{r}{n} \right) a_1 \right)} g$
para-Sasakian	$\frac{(-1)^\ell (n-1)^\ell a_1}{\left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 + \left(-(n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{(-1)^\ell (n-1)^\ell \left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 + \left(-(n-1) - \frac{r}{n} \right) a_1 \right)} g$
(ε) -para-Sasakian	$\frac{(-\varepsilon)^\ell (n-1)^\ell a_1}{\left(\left(-\varepsilon - \frac{r}{n(n-1)} \right) a_0 + \left(-\varepsilon(n-1) - \frac{r}{n} \right) a_1 \right)} S$ $+ \frac{(-\varepsilon)^\ell (n-1)^\ell \left(\left(-\varepsilon - \frac{r}{n(n-1)} \right) a_0 - \left(\frac{r}{n} \right) a_1 \right)}{\left(\left(-\varepsilon - \frac{r}{n(n-1)} \right) a_0 + \left(-\varepsilon(n-1) - \frac{r}{n} \right) a_1 \right)} g$

Corollary 5.21 *Let M be an n -dimensional (R, \mathcal{P}_*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that $a_0 + (n-1)a_1 \neq 0$. If M is not pseudo-projective semisymmetric, then either M is an Einstein manifold or $r = \frac{kn(n-1)a_0}{a_0 + (n-1)a_1}$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:*

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$S = k(n-1)g$ or $r = \frac{kn(n-1)a_0}{a_0 + (n-1)a_1}$
Sasakian	$\frac{1}{n-1}$	$S = (n-1)g$ or $r = \frac{n(n-1)a_0}{a_0 + (n-1)a_1}$
Kenmotsu	$\frac{1}{n-1}$	$S = -(n-1)g$ or $r = -\frac{n(n-1)a_0}{a_0 + (n-1)a_1}$
(ε) -Sasakian	$\frac{1}{n-1}$	$S = \varepsilon(n-1)g$ or $r = \frac{\varepsilon n(n-1)a_0}{a_0 + (n-1)a_1}$
para-Sasakian	$\frac{1}{n-1}$	$S = -(n-1)g$ or $r = -\frac{n(n-1)a_0}{a_0 + (n-1)a_1}$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$S = -\varepsilon(n-1)g$ or $r = -\frac{\varepsilon n(n-1)a_0}{a_0 + (n-1)a_1}$

Corollary 5.22 *Let M be an n -dimensional (R, \mathcal{P}, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not projective semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$. Consequently, we have the following:*

M	$S =$
$N(k)$ -contact metric	$k(n-1)g$
Sasakian	$(n-1)g$
Kenmotsu	$-(n-1)g$
(ε) -Sasakian	$\varepsilon(n-1)g$
para-Sasakian	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon(n-1)g$

Corollary 5.23 *Let M be an n -dimensional (R, \mathcal{P}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not projective semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:*

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu [16]	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.24 *Let M be an n -dimensional (R, \mathcal{M}, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{M} -semisymmetric, then*

$$S^{\ell+1} = k(n-1)S^\ell + k^\ell(n-1)^\ell S - k^{\ell+1}(n-1)^{\ell+1}g.$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$k(n-1)S^\ell + k^\ell(n-1)^\ell S - k^{\ell+1}(n-1)^{\ell+1}g$
Sasakian	$(n-1)S^\ell + (n-1)^\ell S - (n-1)^{\ell+1}g$
Kenmotsu	$-(n-1)S^\ell + (-1)^\ell(n-1)^\ell S - (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -Sasakian	$\varepsilon(n-1)S^\ell + (\varepsilon)^\ell(n-1)^\ell S - (\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$
para-Sasakian	$-(n-1)S^\ell + (-1)^\ell(n-1)^\ell S - (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -para-Sasakian	$-\varepsilon(n-1)S^\ell + (-\varepsilon)^\ell(n-1)^\ell S - (-\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$

Corollary 5.25 *Let M be an n -dimensional (R, \mathcal{M}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{M} -semisymmetric, then*

$$S^2 = 2(n-1)kS - k^2(n-1)^2g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$2(n-1)kS - k^2(n-1)^2g$
Sasakian	$2(n-1)S - (n-1)^2g$
Kenmotsu	$-2(n-1)S - (n-1)^2g$
(ε) -Sasakian	$2(n-1)\varepsilon S - (n-1)^2g$
para-Sasakian	$-2(n-1)S - (n-1)^2g$
(ε) -para-Sasakian	$-2(n-1)\varepsilon S - (n-1)^2g$

Corollary 5.26 *Let M be an n -dimensional $(R, \mathcal{W}_0, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_0 -semisymmetric, then*

$$S^{\ell+1} = k(n-1)S^\ell + k^\ell(n-1)^\ell S - k^{\ell+1}(n-1)^{\ell+1}g.$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$k(n-1)S^\ell + k^\ell(n-1)^\ell S - k^{\ell+1}(n-1)^{\ell+1}g$
Sasakian	$(n-1)S^\ell + (n-1)^\ell S - (n-1)^{\ell+1}g$
Kenmotsu	$-(n-1)S^\ell + (-1)^\ell(n-1)^\ell S - (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -Sasakian	$\varepsilon(n-1)S^\ell + (\varepsilon)^\ell(n-1)^\ell S - (\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$
para-Sasakian	$-(n-1)S^\ell + (-1)^\ell(n-1)^\ell S - (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -para-Sasakian	$-\varepsilon(n-1)S^\ell + (-\varepsilon)^\ell(n-1)^\ell S - (-\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$

Corollary 5.27 *Let M be an n -dimensional (R, \mathcal{W}_0, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_0 -semisymmetric, then*

$$S^2 = 2(n-1)kS - k^2(n-1)^2g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$2(n-1)kS - k^2(n-1)^2g$
Sasakian	$2(n-1)S - (n-1)^2g$
Kenmotsu	$-2(n-1)S - (n-1)^2g$
(ε) -Sasakian	$2(n-1)\varepsilon S - (n-1)^2g$
para-Sasakian	$-2(n-1)S - (n-1)^2g$
(ε) -para-Sasakian	$-2(n-1)\varepsilon S - (n-1)^2g$

Corollary 5.28 *Let M be an n -dimensional $(R, \mathcal{W}_0^*, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_0^* -semisymmetric, then*

$$S^{\ell+1} = -k(n-1)S^\ell + k^\ell(n-1)^\ell S + k^{\ell+1}(n-1)^{\ell+1}g.$$

Consequently, we have the following:

M	$S^{\ell+1} =$
$N(k)$ -contact metric	$-k(n-1)S^\ell + k^\ell(n-1)^\ell S + k^{\ell+1}(n-1)^{\ell+1}g$
Sasakian	$-(n-1)S^\ell + (n-1)^\ell S + (n-1)^{\ell+1}g$
Kenmotsu	$(n-1)S^\ell + (-1)^\ell(n-1)^\ell S + (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -Sasakian	$-\varepsilon(n-1)S^\ell + (\varepsilon)^\ell(n-1)^\ell S + (\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$
para-Sasakian	$(n-1)S^\ell + (-1)^\ell(n-1)^\ell S + (-1)^{\ell+1}(n-1)^{\ell+1}g$
(ε) -para-Sasakian	$\varepsilon(n-1)S^\ell + (-\varepsilon)^\ell(n-1)^\ell S + (-\varepsilon)^{\ell+1}(n-1)^{\ell+1}g$

Corollary 5.29 Let M be an n -dimensional (R, \mathcal{W}_0^*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_0^* -semisymmetric, then

$$S^2 = k^2(n-1)^2g.$$

Consequently, we have the following:

M	$S^2 =$
$N(k)$ -contact metric	$k^2(n-1)^2g$
Sasakian	$(n-1)^2g$
Kenmotsu	$(n-1)^2g$
(ε) -Sasakian	$(n-1)^2g$
para-Sasakian	$(n-1)^2g$
(ε) -para-Sasakian	$(n-1)^2g$

Corollary 5.30 Let M be an n -dimensional $(R, \mathcal{W}_1, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_1 -semisymmetric, then

$$2S^\ell = k^{\ell-1}(n-1)^{\ell-1}S + k^\ell(n-1)^\ell g.$$

Consequently, we have the following:

M	$2S^\ell =$
$N(k)$ -contact metric	$k^{\ell-1}(n-1)^{\ell-1}S + k^\ell(n-1)^\ell g$
Sasakian	$(n-1)^{\ell-1}S + (n-1)^\ell g$
Kenmotsu	$(-1)^{\ell-1}(n-1)^{\ell-1}S + (-1)^\ell(n-1)^\ell g$
(ε) -Sasakian	$(\varepsilon)^{\ell-1}(n-1)^{\ell-1}S + (\varepsilon)^\ell(n-1)^\ell g$
para-Sasakian	$(-1)^{\ell-1}(n-1)^{\ell-1}S + (-1)^\ell(n-1)^\ell g$
(ε) -para-Sasakian	$(-\varepsilon)^{\ell-1}(n-1)^{\ell-1}S + (-\varepsilon)^\ell(n-1)^\ell g$

Corollary 5.31 Let M be an n -dimensional (R, \mathcal{W}_1, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_1 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.32 Let M be an n -dimensional $(R, \mathcal{W}_1^*, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_1^* -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$. Consequently, we have the following:

M	$S =$
$N(k)$ -contact metric	$k(n-1)g$
Sasakian	$(n-1)g$
Kenmotsu	$-(n-1)g$
(ε) -Sasakian	$\varepsilon(n-1)g$
para-Sasakian	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon(n-1)g$

Corollary 5.33 *Let M be an n -dimensional (R, \mathcal{W}_1^*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_1^* -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:*

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.34 *Let M be an n -dimensional $(R, \mathcal{W}_2, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_2 -semisymmetric, then*

$$L_{S^\ell} S^{\ell+1} = k(n-1)L_{S^\ell} S^\ell + kS - k^2(n-1)g.$$

Consequently, we have the following:

M	$L_{S^\ell} S^{\ell+1} =$
$N(k)$ -contact metric	$k(n-1)L_{S^\ell} S^\ell + kS - k^2(n-1)g$
Sasakian	$(n-1)L_{S^\ell} S^\ell + S - (n-1)g$
Kenmotsu	$-(n-1)L_{S^\ell} S^\ell - S - (n-1)g$
(ε) -Sasakian	$\varepsilon(n-1)L_{S^\ell} S^\ell + \varepsilon S - (n-1)g$
para-Sasakian	$-(n-1)L_{S^\ell} S^\ell - S - (n-1)g$
(ε) -para-Sasakian	$-\varepsilon(n-1)L_{S^\ell} S^\ell - \varepsilon S - (n-1)g$

Corollary 5.35 *Let M be an n -dimensional (R, \mathcal{W}_2, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_2 -semisymmetric, then*

$$L_S S^2 = k((n-1)L_S + 1)S - k^2(n-1)g.$$

Consequently, we have the following:

M	$L_S S^2 =$
$N(k)$ -contact metric	$k((n-1)L_S + 1)S - k^2(n-1)g$
Sasakian	$((n-1)L_S + 1)S - (n-1)g$
Kenmotsu	$-((n-1)L_S + 1)S - (n-1)g$
(ε) -Sasakian	$\varepsilon((n-1)L_S + 1)S - (n-1)g$
para-Sasakian	$-((n-1)L_S + 1)S - (n-1)g$
(ε) -para-Sasakian	$-\varepsilon((n-1)L_S + 1)S - (n-1)g$

Corollary 5.36 *Let M be an n -dimensional $(R, \mathcal{W}_3, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_3 -semisymmetric, then*

$$S^\ell = k^\ell(n-1)^\ell g$$

and $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	$S^\ell =$
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$k^\ell(n-1)^\ell g$
Sasakian	$\frac{1}{(n-1)^\ell}$	$(n-1)^\ell g$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -Sasakian	$\frac{1}{(\varepsilon)^{\ell-1}(n-1)^\ell}$	$(\varepsilon)^\ell(n-1)^\ell g$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$(-\varepsilon)^\ell(n-1)^\ell g$

Corollary 5.37 Let M be an n -dimensional (R, \mathcal{W}_3, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_3 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.38 Let M be an n -dimensional $(R, \mathcal{W}_4, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_4 -semisymmetric, then

$$L_{S^\ell} S^{\ell+1} = k(n-1)L_{S^\ell} S^\ell + kS - k^2(n-1)g.$$

Consequently, we have the following:

M	$L_{S^\ell} S^{\ell+1} =$
$N(k)$ -contact metric	$k(n-1)L_{S^\ell} S^\ell + kS - k^2(n-1)g$
Sasakian	$(n-1)L_{S^\ell} S^\ell + S - (n-1)g$
Kenmotsu	$-(n-1)L_{S^\ell} S^\ell - S - (n-1)g$
(ε) -Sasakian	$\varepsilon(n-1)L_{S^\ell} S^\ell + \varepsilon S - (n-1)g$
para-Sasakian	$-(n-1)L_{S^\ell} S^\ell - S - (n-1)g$
(ε) -para-Sasakian	$-\varepsilon(n-1)L_{S^\ell} S^\ell - \varepsilon S - (n-1)g$

Corollary 5.39 Let M be an n -dimensional (R, \mathcal{W}_4, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_4 -semisymmetric, then

$$L_S S^2 = k((n-1)L_S + 1)S - k^2(n-1)g.$$

Consequently, we have the following:

M	$L_S S^2 =$
$N(k)$ -contact metric	$k((n-1)L_S + 1)S - k^2(n-1)g$
Sasakian	$((n-1)L_S + 1)S - (n-1)g$
Kenmotsu	$-((n-1)L_S + 1)S - (n-1)g$
(ε) -Sasakian	$\varepsilon((n-1)L_S + 1)S - (n-1)g$
para-Sasakian	$-((n-1)L_S + 1)S - (n-1)g$
(ε) -para-Sasakian	$-\varepsilon((n-1)L_S + 1)S - (n-1)g$

Corollary 5.40 *Let M be an n -dimensional $(R, \mathcal{W}_5, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_5 -semisymmetric, then either*

$$S = k(n-1)g$$

or $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	$S =$
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$k(n-1)g$
Sasakian	$\frac{1}{(n-1)^\ell}$	$(n-1)g$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$-\varepsilon(n-1)g$

Corollary 5.41 *Let M be an n -dimensional (R, \mathcal{W}_5, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_5 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and*

$$L_S = \frac{1}{n-1}. \text{ Consequently, we have the following:}$$

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.42 *Let M be an n -dimensional $(R, \mathcal{W}_6, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_6 -semisymmetric, then*

$$(n-1)L_{S^\ell}S^\ell = (1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g.$$

Consequently, we have the following:

M	$(n-1)L_{S^\ell}S^\ell =$
$N(k)$ -contact metric	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
Sasakian	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
Kenmotsu	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
(ε) -Sasakian	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
para-Sasakian	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
(ε) -para-Sasakian	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$

Corollary 5.43 Let M be an n -dimensional (R, \mathcal{W}_6, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_6 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{2(n-1)}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{2(n-1)}$	$k(n-1)g$
Sasakian	$\frac{1}{2(n-1)}$	$(n-1)g$
Kenmotsu	$\frac{1}{2(n-1)}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{2(n-1)}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{2(n-1)}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{2(n-1)}$	$-\varepsilon(n-1)g$

Corollary 5.44 Let M be an n -dimensional $(R, \mathcal{W}_7, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_7 -semisymmetric, then

$$(n-1)L_{S^\ell}S^\ell = (1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g.$$

Consequently, we have the following:

M	$(n-1)L_{S^\ell}S^\ell =$
$N(k)$ -contact metric	$(1 - k^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2k^\ell(n-1)^{\ell+1}L_{S^\ell} - k(n-1))g$
Sasakian	$(1 - (n-1)^\ell L_{S^\ell})S + (2(n-1)^{\ell+1}L_{S^\ell} - (n-1))g$
Kenmotsu	$(1 - (-1)^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2(-1)^\ell(n-1)^{\ell+1}L_{S^\ell} + (n-1))g$
(ε) -Sasakian	$(1 - (\varepsilon)^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2(\varepsilon)^\ell(n-1)^{\ell+1}L_{S^\ell} - \varepsilon(n-1))g$
para-Sasakian	$(1 - (-1)^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2(-1)^\ell(n-1)^{\ell+1}L_{S^\ell} + (n-1))g$
(ε) -para-Sasakian	$(1 - (-\varepsilon)^{\ell-1}(n-1)^\ell L_{S^\ell})S + (2(-\varepsilon)^\ell(n-1)^{\ell+1}L_{S^\ell} + \varepsilon(n-1))g$

Corollary 5.45 Let M be an n -dimensional (R, \mathcal{W}_7, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_7 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{2(n-1)}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{2(n-1)}$	$k(n-1)g$
Sasakian	$\frac{1}{2(n-1)}$	$(n-1)g$
Kenmotsu	$\frac{1}{2(n-1)}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{2(n-1)}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{2(n-1)}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{2(n-1)}$	$-\varepsilon(n-1)g$

Corollary 5.46 Let M be an n -dimensional $(R, \mathcal{W}_8, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_8 -semisymmetric, then

$$S^\ell = k^\ell(n-1)^\ell g$$

and $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	$S^\ell =$
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$k^\ell(n-1)^\ell g$
Sasakian	$\frac{1}{(n-1)^\ell}$	$(n-1)^\ell g$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -Sasakian	$\frac{1}{(\varepsilon)^{\ell-1}(n-1)^\ell}$	$(\varepsilon)^\ell(n-1)^\ell g$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$(-\varepsilon)^\ell(n-1)^\ell g$

Corollary 5.47 Let M be an n -dimensional (R, \mathcal{W}_8, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_8 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and $L_S = \frac{1}{n-1}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

Corollary 5.48 Let M be an n -dimensional $(R, \mathcal{W}_9, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_9 -semisymmetric, then

$$S^\ell = k^\ell(n-1)^\ell g$$

and $L_{S^\ell} = \frac{1}{k^{\ell-1}(n-1)^\ell}$. Consequently, we have the following:

M	$L_{S^\ell} =$	$S^\ell =$
$N(k)$ -contact metric	$\frac{1}{k^{\ell-1}(n-1)^\ell}$	$k^\ell(n-1)^\ell g$
Sasakian	$\frac{1}{(n-1)^\ell}$	$(n-1)^\ell g$
Kenmotsu	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -Sasakian	$\frac{1}{(\varepsilon)^{\ell-1}(n-1)^\ell}$	$(\varepsilon)^\ell(n-1)^\ell g$
para-Sasakian	$\frac{1}{(-1)^{\ell-1}(n-1)^\ell}$	$(-1)^\ell(n-1)^\ell g$
(ε) -para-Sasakian	$\frac{1}{(-\varepsilon)^{\ell-1}(n-1)^\ell}$	$(-\varepsilon)^\ell(n-1)^\ell g$

Corollary 5.49 Let M be an n -dimensional (R, \mathcal{W}_9, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. If M is not \mathcal{W}_9 -semisymmetric, then M is an Einstein manifold with scalar curvature $kn(n-1)$ and

$L_S = \frac{1}{n-1}$. Consequently, we have the following:

M	$L_S =$	$S =$
$N(k)$ -contact metric	$\frac{1}{n-1}$	$k(n-1)g$
Sasakian	$\frac{1}{n-1}$	$(n-1)g$
Kenmotsu	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -Sasakian	$\frac{1}{n-1}$	$\varepsilon(n-1)g$
para-Sasakian	$\frac{1}{n-1}$	$-(n-1)g$
(ε) -para-Sasakian	$\frac{1}{n-1}$	$-\varepsilon(n-1)g$

6 $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetry

In this section, we determine the results for an n -dimensional $(N(k), \xi)$ -semi-Riemannian manifold satisfy $\mathcal{T}_a \cdot S_{\mathcal{T}_b} = LQ(g, S_{\mathcal{T}_b})$.

Definition 6.1 A semi-Riemannian manifold is said to be $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric if

$$\mathcal{T}_a \cdot S_{\mathcal{T}_b} = LQ(g, S_{\mathcal{T}_b}), \quad (6.1)$$

where L is some smooth function defined on M . In particular, it is said to be $(R \cdot S_{\mathcal{T}_a})$ -pseudosymmetric if it satisfies

$$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a}), \quad (6.2)$$

holds on the set $\mathcal{U} = \left\{ x \in M : \left(S_{\mathcal{T}_a} - \frac{\text{tr}(S_{\mathcal{T}_a})}{n}g \right)_x \neq 0 \right\}$, where L is some function defined on \mathcal{U} .

Remark 6.2 If in (6.2), $S_{\mathcal{T}_a}$ is replaced by S then it is said to be Ricci-pseudosymmetric.

Theorem 6.3 Let M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& \varepsilon a_5(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)S^2(Y, U) \\
& + \{ \varepsilon(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(b_4r + (n-1)b_7r) \} S(Y, U) \\
& + \{ \varepsilon k(n-1)(a_2 + a_4)(b_4r + (n-1)b_7r) \\
& + \varepsilon k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r) \} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \times \\
& \{ b_4r + (n-1)b_7r \\
& + k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \} \eta(Y)\eta(U) \\
& = L(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)(\varepsilon k(n-1)g(Y, U) - \varepsilon S(Y, U)).
\end{aligned}$$

In particular, if M is an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_a})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold,

then

$$\begin{aligned}
& \varepsilon a_5(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)S^2(Y, U) \\
& + \{ \varepsilon(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(a_4r + (n-1)a_7r) \} S(Y, U) \\
& + \{ \varepsilon k(n-1)(a_2 + a_4)(a_4r + (n-1)a_7r) \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r) \} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \\
& \{ a_4r + (n-1)a_7r \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \} \eta(Y)\eta(U) \\
= & L(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)(\varepsilon k(n-1)g(Y, U) - \varepsilon S(Y, U)).
\end{aligned}$$

Proof. Let M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_b})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\mathcal{T}_a(X, Y) \cdot S_{\mathcal{T}_b}(U, V) = LQ(g, S_{\mathcal{T}_b})(U, V; X, Y). \quad (6.3)$$

Taking $X = \xi = V$ in (6.3), we have

$$\mathcal{T}_a(\xi, Y) \cdot S_{\mathcal{T}_b}(U, \xi) = LQ(g, S_{\mathcal{T}_b})(U, \xi; \xi, Y),$$

which gives

$$\begin{aligned}
& S_{\mathcal{T}_b}(\mathcal{T}_a(\xi, Y)U, \xi) + S_{\mathcal{T}_b}(U, \mathcal{T}_a(\xi, Y)\xi) \\
= & L(S_{\mathcal{T}_b}((\xi \wedge Y)U, \xi) + S_{\mathcal{T}_b}(U, (\xi \wedge Y)\xi)).
\end{aligned} \quad (6.4)$$

Using (3.1), (3.23), (3.30), (3.31), (3.35) and (3.36) in (6.4), we get the result. ■

Theorem 6.4 *Let M be an n -dimensional (\mathcal{T}_a, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& \varepsilon a_5 S^2(Y, U) - E S(Y, U) - F g(Y, U) - G \eta(Y) \eta(U) \\
= & L(\varepsilon k(n-1)g(Y, U) - \varepsilon S(Y, U)),
\end{aligned}$$

where

$$\begin{aligned}
E &= \varepsilon(ka_0 + a_7r - k(n-1)a_1 - k(n-1)a_2), \\
F &= -\varepsilon k(n-1)(ka_0 + k(n-1)a_4 + a_7r), \\
G &= -k^2(n-1)^2(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6).
\end{aligned}$$

In view of Theorem 6.4, we have the following

Corollary 6.5 *Let M be an n -dimensional Ricci-pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$L =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 6.6 Let M be an n -dimensional (\mathcal{C}_*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$-\left(\left(k - \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} - \frac{2r}{n} - \frac{L}{a_1}\right) S$ $+k(n-1) \left(\left(k - \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + k(n-1) - \frac{2r}{n} - \frac{L}{a_1}\right) g$
Sasakian	$-\left(\left(1 - \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} - \frac{2r}{n} - \frac{L}{a_1}\right) S$ $+(n-1) \left(\left(1 - \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + (n-1) - \frac{2r}{n} - \frac{L}{a_1}\right) g$
Kenmotsu	$\left(\left(1 + \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + \frac{2r}{n} + \frac{L}{a_1}\right) S$ $+(n-1) \left(\left(1 + \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + (n-1) + \frac{2r}{n} + \frac{L}{a_1}\right) g$
(ε) -Sasakian	$-\varepsilon \left(\left(1 - \frac{\varepsilon r}{n(n-1)}\right) \frac{a_0}{a_1} - \frac{2\varepsilon r}{n} - \frac{\varepsilon L}{a_1}\right) S$ $+\varepsilon(n-1) \left(\left(\varepsilon - \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + \varepsilon(n-1) - \frac{2r}{n} - \frac{L}{a_1}\right) g$
para-Sasakian	$\left(\left(1 + \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + \frac{2r}{n} + \frac{L}{a_1}\right) S$ $+(n-1) \left(\left(1 + \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + (n-1) + \frac{2r}{n} + \frac{L}{a_1}\right) g$
(ε) -para-Sasakian	$\varepsilon \left(\left(1 + \frac{\varepsilon r}{n(n-1)}\right) \frac{a_0}{a_1} + \frac{2\varepsilon r}{n} + \frac{\varepsilon L}{a_1}\right) S$ $+\varepsilon(n-1) \left(\left(\varepsilon + \frac{r}{n(n-1)}\right) \frac{a_0}{a_1} + \varepsilon(n-1) + \frac{2r}{n} + \frac{L}{a_1}\right) g$

Corollary 6.7 Let M be an n -dimensional (\mathcal{C}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$\left(\frac{r}{n-1} + (k-L)(n-2)\right) S - k(r - (L(n-2) + k)(n-1))g$
Sasakian	$\left(\frac{r}{n-1} + (1-L)(n-2)\right) S - (r - (L(n-2) + 1)(n-1))g$
Kenmotsu	$\left(\frac{r}{n-1} - (1+L)(n-2)\right) S + (r - (L(n-2) - 1)(n-1))g$
(ε) -Sasakian	$\left(\frac{r}{n-1} + (\varepsilon-L)(n-2)\right) S - \varepsilon(r - (L(n-2) + \varepsilon)(n-1))g$
para-Sasakian	$\left(\frac{r}{n-1} - (1+L)(n-2)\right) S + 1(r - (L(n-2) - 1)(n-1))g$
(ε) -para-Sasakian	$\left(\frac{r}{n-1} - (\varepsilon+L)(n-2)\right) S + \varepsilon(r - (L(n-2) - \varepsilon)(n-1))g$

Corollary 6.8 Let M be an n -dimensional (\mathcal{L}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$(n-2)(k-L)S + k(n-1)(k + (n-2)L)g$
Sasakian	$(n-2)(1-L)S + (n-1)(1 + (n-2)L)g$
Kenmotsu	$-(n-2)(1+L)S - (n-1)(-1 + (n-2)L)g$
(ε) -Sasakian	$(n-2)(\varepsilon-L)S + \varepsilon(n-1)(\varepsilon + (n-2)L)g$
para-Sasakian	$-(n-2)(1+L)S - (n-1)(-1 + (n-2)L)g$
(ε) -para-Sasakian	$-(n-2)(\varepsilon+L)S - \varepsilon(n-1)(-\varepsilon + (n-2)L)g$

Corollary 6.9 Let M be an n -dimensional (\mathcal{V}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	$k - \frac{r}{n(n-1)}$	$k(n-1)g$
Sasakian	$1 - \frac{r}{n(n-1)}$	$(n-1)g$
Kenmotsu	$-1 - \frac{r}{n(n-1)}$	$-(n-1)g$
(ε) -Sasakian	$\varepsilon - \frac{r}{n(n-1)}$	$\varepsilon(n-1)g$
para-Sasakian	$-1 - \frac{r}{n(n-1)}$	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon - \frac{r}{n(n-1)}$	$-\varepsilon(n-1)g$

Corollary 6.10 Let M be an n -dimensional (\mathcal{P}_*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	$\left(k - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$k(n-1)g$
Sasakian	$\left(1 - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$(n-1)g$
Kenmotsu	$\left(-1 - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$-(n-1)g$
(ε) -Sasakian	$\left(\varepsilon - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$\varepsilon(n-1)g$
para-Sasakian	$\left(-1 - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$-(n-1)g$
(ε) -para-Sasakian	$\left(-\varepsilon - \frac{r}{n(n-1)}\right) a_0 - \frac{r}{n} a_1$	$-\varepsilon(n-1)g$

Corollary 6.11 Let M be an n -dimensional (\mathcal{P}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 6.12 Let M be an n -dimensional (\mathcal{M}, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$2(n-1)(k-L)S - k(n-1)^2(k-2L)g$
Sasakian	$2(n-1)(1-L)S - (n-1)^2(1-2L)g$
Kenmotsu	$-2(n-1)(1+L)S - (n-1)^2(1+2L)g$
(ε) -Sasakian	$2(n-1)(\varepsilon-L)S - \varepsilon(n-1)^2(\varepsilon-2L)g$
para-Sasakian	$-2(n-1)(1+L)S - (n-1)^2(1+2L)g$
(ε) -para-Sasakian	$-2(n-1)(\varepsilon+L)S - \varepsilon(n-1)^2(\varepsilon+2L)g$

Corollary 6.13 Let M be an n -dimensional (\mathcal{W}_0, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

fold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$(n-1)(2k-L)S + k(n-1)^2(L-k)g$
Sasakian	$(n-1)(2-L)S + (n-1)^2(L-1)g$
Kenmotsu	$-(n-1)(2+L)S - (n-1)^2(L+1)g$
(ε) -Sasakian	$(n-1)(2\varepsilon-L)S + \varepsilon(n-1)^2(L-\varepsilon)g$
para-Sasakian	$-(n-1)(2+L)S - (n-1)^2(L+1)g$
(ε) -para-Sasakian	$-(n-1)(2\varepsilon+L)S - \varepsilon(n-1)^2(L+\varepsilon)g$

Corollary 6.14 Let M be an n -dimensional (\mathcal{W}_0^*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$L(n-1)S + k(n-1)^2(k-L)g$
Sasakian	$L(n-1)S + (n-1)^2(1-L)g$
Kenmotsu	$L(n-1)S + (n-1)^2(1+L)g$
(ε) -Sasakian	$L(n-1)S + \varepsilon(n-1)^2(\varepsilon-L)g$
para-Sasakian	$L(n-1)S + (n-1)^2(1+L)g$
(ε) -para-Sasakian	$L(n-1)S + \varepsilon(n-1)^2(\varepsilon+L)g$

Corollary 6.15 Let M be an n -dimensional (\mathcal{W}_1, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 6.16 Let M be an n -dimensional (\mathcal{W}_1^*, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	k	$k(n-1)g$
Sasakian	1	$(n-1)g$
Kenmotsu	-1	$-(n-1)g$
(ε) -Sasakian	ε	$\varepsilon(n-1)g$
para-Sasakian	-1	$-(n-1)g$
(ε) -para-Sasakian	$-\varepsilon$	$-\varepsilon(n-1)g$

Corollary 6.17 Let M be an n -dimensional (\mathcal{W}_2, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$(n-1)(k-L)S + k(n-1)^2Lg$
Sasakian	$(n-1)(1-L)S + (n-1)^2Lg$
Kenmotsu	$-(n-1)(1+L)S - (n-1)^2Lg$
(ε) -Sasakian	$(n-1)(\varepsilon-L)S + \varepsilon(n-1)^2Lg$
para-Sasakian	$-(n-1)(1+L)S - (n-1)^2Lg$
(ε) -para-Sasakian	$-(n-1)(\varepsilon+L)S - \varepsilon(n-1)^2Lg$

Corollary 6.18 Let M be an n -dimensional (\mathcal{W}_3, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

fold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	$2k$	$k(n-1)g$
Sasakian	2	$(n-1)g$
Kenmotsu	-2	$-(n-1)g$
(ε) -Sasakian	2ε	$\varepsilon(n-1)g$
para-Sasakian	-2	$-(n-1)g$
(ε) -para-Sasakian	-2ε	$-\varepsilon(n-1)g$

Corollary 6.19 Let M be an n -dimensional (\mathcal{W}_4, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$(n-1)(k-L)S + k(n-1)^2(L-k)g + \varepsilon k^2(n-1)^2\eta \otimes \eta$
Sasakian	$(n-1)(1-L)S + (n-1)^2(L-1)g + \varepsilon(n-1)^2\eta \otimes \eta$
Kenmotsu	$-(n-1)(1+L)S - (n-1)^2(L+1)g + \varepsilon(n-1)^2\eta \otimes \eta$
(ε) -Sasakian	$(n-1)(\varepsilon-L)S + \varepsilon(n-1)^2(L-\varepsilon)g + \varepsilon(n-1)^2\eta \otimes \eta$
para-Sasakian	$-(n-1)(1+L)S - (n-1)^2(L+1)g + \varepsilon(n-1)^2\eta \otimes \eta$
(ε) -para-Sasakian	$-(n-1)(\varepsilon+L)S - \varepsilon(n-1)^2(L+\varepsilon)g + \varepsilon(n-1)^2\eta \otimes \eta$

Corollary 6.20 Let M be an n -dimensional (\mathcal{W}_5, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$S^2 =$
$N(k)$ -contact metric	$(n-1)(2k-L)S + k(n-1)^2(L-k)g$
Sasakian	$(n-1)(2-L)S + (n-1)^2(L-1)g$
Kenmotsu	$-(n-1)(2+L)S - (n-1)^2(L+1)g$
(ε) -Sasakian	$(n-1)(2\varepsilon-L)S + \varepsilon(n-1)^2(L-\varepsilon)g$
para-Sasakian	$-(n-1)(2+L)S - (n-1)^2(L+1)g$
(ε) -para-Sasakian	$-(n-1)(2\varepsilon+L)S + k(n-1)^2(L+\varepsilon)g$

Corollary 6.21 Let M be an n -dimensional (\mathcal{W}_6, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	Result
$N(k)$ -contact metric	$(2k-L)S + k(n-1)(L-k)g + k^2(n-1)\eta \otimes \eta = 0$
Sasakian	$(2-L)S + (n-1)(L-1)g + (n-1)\eta \otimes \eta = 0$
Kenmotsu	$-(2+L)S - (n-1)(L+1)g + (n-1)\eta \otimes \eta = 0$
(ε) -Sasakian	$(2\varepsilon-L)S + \varepsilon(n-1)(L-\varepsilon)g + \varepsilon(n-1)\eta \otimes \eta = 0$
para-Sasakian	$-(2+L)S - (n-1)(L+1)g + (n-1)\eta \otimes \eta = 0$
(ε) -para-Sasakian	$-(2\varepsilon+L)S - \varepsilon(n-1)(L+\varepsilon)g + \varepsilon(n-1)\eta \otimes \eta = 0$

Corollary 6.22 Let M be an n -dimensional (\mathcal{W}_7, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L =$	$S =$
$N(k)$ -contact metric	$2k$	$k(n-1)g$
Sasakian	2	$(n-1)g$
Kenmotsu	-2	$-(n-1)g$
(ε) -Sasakian	2ε	$\varepsilon(n-1)g$
para-Sasakian	-2	$-(n-1)g$
(ε) -para-Sasakian	-2ε	$-\varepsilon(n-1)g$

Corollary 6.23 Let M be an n -dimensional (\mathcal{W}_8, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

fold. Then we have the following table:

M	Result
$N(k)$ -contact metric	$(2k - L)S + k(n - 1)(L - k)g - k^2(n - 1)\eta \otimes \eta = 0$
Sasakian	$(2 - L)S + (n - 1)(L - 1)g - (n - 1)\eta \otimes \eta = 0$
Kenmotsu	$-(2 + L)S - (n - 1)(L + 1)g - (n - 1)\eta \otimes \eta = 0$
(ε) -Sasakian	$(2\varepsilon - L)S + \varepsilon(n - 1)(L - \varepsilon)g - \varepsilon(n - 1)\eta \otimes \eta = 0$
para-Sasakian	$-(2 + L)S - (n - 1)(L + 1)g - (n - 1)\eta \otimes \eta = 0$
(ε) -para-Sasakian	$-(2\varepsilon + L)S - \varepsilon(n - 1)(L + \varepsilon)g - \varepsilon(n - 1)\eta \otimes \eta = 0$

Corollary 6.24 Let M be an n -dimensional (\mathcal{W}_9, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	Result
$N(k)$ -contact metric	$(L - k)S - k(n - 1)Lg + k^2(n - 1)\eta \otimes \eta = 0$
Sasakian	$(L - 1)S - (n - 1)Lg + (n - 1)\eta \otimes \eta = 0$
Kenmotsu	$(L + 1)S + (n - 1)Lg + (n - 1)\eta \otimes \eta = 0$
(ε) -Sasakian	$(L - \varepsilon)S - \varepsilon(n - 1)Lg + \varepsilon(n - 1)\eta \otimes \eta = 0$
para-Sasakian	$(L + 1)S + (n - 1)Lg + (n - 1)\eta \otimes \eta = 0$
(ε) -para-Sasakian	$(L + \varepsilon)S + \varepsilon(n - 1)Lg + \varepsilon(n - 1)\eta \otimes \eta = 0$

Theorem 6.25 Let M be an n -dimensional $(R, S_{\mathcal{T}_a})$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold such that

$$a_0 + na_1 + a_2 + a_3 + a_5 + a_6 \neq 0.$$

Then M is either Einstein manifold, that is,

$$S = k(n - 1)g$$

or $L = k$ holds on M . Consequently, we have the following table:

Manifold	Condition	$S =$	$L =$
$N(k)$ -contact metric	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$k(n - 1)g$	k
Sasakian	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$(n - 1)g$	1
Kenmotsu	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$-(n - 1)g$	-1
(ε) -Sasakian	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$\varepsilon(n - 1)g$	ε
para-Sasakian	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$-(n - 1)g$	-1
(ε) -para-Sasakian	$R \cdot S_{\mathcal{T}_a} = LQ(g, S_{\mathcal{T}_a})$	$-\varepsilon(n - 1)g$	$-\varepsilon$

Remark 6.26 The conclusions of Theorem 6.25 remain true if $S_{\mathcal{T}_a}$ is replaced by S .

Corollary 6.27 ([16], [11]) If an n -dimensional Kenmotsu manifold M is Ricci-pseudosymmetric then either M is an Einstein manifold with the scalar curvature $r = n(1 - n)$ or $L = -1$ holds on M .

7 $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetry

In this section, we determine the result for an n -dimensional $(N(k), \xi)$ -semi-Riemannian manifold satisfy $\mathcal{T}_a \cdot S_{\mathcal{T}_b} = LQ(S^\ell, S_{\mathcal{T}_b})$.

Definition 7.1 A semi-Riemannian manifold M is called $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric if

$$\mathcal{T}_a \cdot S_{\mathcal{T}_b} = LQ(S^\ell, S_{\mathcal{T}_b}),$$

where L is some smooth function defined on M . In particular, M is said to be $(R, S_{\mathcal{T}_a}, S^\ell)$ -pseudosymmetric if

$$R \cdot S_{\mathcal{T}_a} = LQ(S^\ell, S_{\mathcal{T}_a}).$$

Theorem 7.2 *Let M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& \varepsilon a_5(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)S^2(Y, U) \\
& + \{\varepsilon(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(b_4r + (n-1)b_7r)\} S(Y, U) \\
& + \{\varepsilon k(n-1)(a_2 + a_4)(b_4r + (n-1)b_7r) \\
& + \varepsilon k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r)\} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \times \\
& \{b_4r + (n-1)b_7r \\
& + k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)\} \eta(Y)\eta(U) \\
= & L\varepsilon((b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (k(n-1)S^\ell(Y, U) - k^\ell(n-1)^\ell S(Y, U)) \\
& + (b_4 + (n-1)b_7r)(k^\ell(n-1)^\ell g(Y, U) - S^\ell(Y, U))).
\end{aligned}$$

In particular, if M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_a}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& \varepsilon a_5(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)S^2(Y, U) \\
& + \{\varepsilon(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(a_4r + (n-1)a_7r)\} S(Y, U) \\
& + \{\varepsilon k(n-1)(a_2 + a_4)(a_4r + (n-1)a_7r) \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r)\} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \\
& \{a_4r + (n-1)a_7r \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)\} \eta(Y)\eta(U) \\
= & L\varepsilon((a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (k(n-1)S^\ell(Y, U) - k^\ell(n-1)^\ell S(Y, U)) \\
& + (a_4 + (n-1)a_7r)(k^\ell(n-1)^\ell g(Y, U) - S^\ell(Y, U))).
\end{aligned}$$

Proof. Let M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_b}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\mathcal{T}_a(X, Y) \cdot S_{\mathcal{T}_b}(U, V) = LQ(S^\ell, S_{\mathcal{T}_b})(U, V; X, Y). \quad (7.1)$$

Taking $X = \xi = V$ in (7.1), we have

$$\mathcal{T}_a(\xi, Y) \cdot S_{\mathcal{T}_b}(U, \xi) = LQ(S^\ell, S_{\mathcal{T}_b})(U, \xi; \xi, Y),$$

which gives

$$\begin{aligned}
& S_{\mathcal{T}_b}(\mathcal{T}_a(\xi, Y)U, \xi) + S_{\mathcal{T}_b}(U, \mathcal{T}_a(\xi, Y)\xi) \\
= & L(S_{\mathcal{T}_b}((\xi \wedge_{S^\ell} Y)U, \xi) + S_{\mathcal{T}_b}(U, (\xi \wedge_{S^\ell} Y)\xi)).
\end{aligned} \quad (7.2)$$

Using (3.1), (3.28), (3.30), (3.31), (3.35) and (3.36) in (7.2), we get the result. ■

For $\ell = 1$, we have the following result.

Corollary 7.3 *Let M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_a}, S)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& \varepsilon a_5(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)S^2(Y, U) \\
& + \{\varepsilon(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(b_4r + (n-1)b_7r)\} S(Y, U) \\
& + \{\varepsilon k(n-1)(a_2 + a_4)(b_4r + (n-1)b_7r) \\
& + \varepsilon k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r)\} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \times \\
& \{b_4r + (n-1)b_7r \\
& + k(n-1)(b_0 + nb_1 + b_2 + b_3 + b_5 + b_6)\} \eta(Y)\eta(U) \\
= & L\varepsilon(b_4 + (n-1)b_7)r(k(n-1)g(Y, U) - S(Y, U)).
\end{aligned}$$

In particular, if M be an n -dimensional $(\mathcal{T}_a, S_{\mathcal{T}_a}, S)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& \varepsilon a_5(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)S^2(Y, U) \\
& + \{\varepsilon(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (-ka_0 + k(n-1)a_1 + k(n-1)a_2 - a_7r) \\
& + \varepsilon(a_1 + a_5)(a_4r + (n-1)a_7r)\} S(Y, U) \\
& + \{\varepsilon k(n-1)(a_2 + a_4)(a_4r + (n-1)a_7r) \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (ka_0 + k(n-1)a_4 + a_7r)\} g(Y, U) \\
& + k(n-1)(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6) \\
& \{a_4r + (n-1)a_7r \\
& + \varepsilon k(n-1)(a_0 + na_1 + a_2 + a_3 + a_5 + a_6)\} \eta(Y)\eta(U) \\
= & L\varepsilon(a_4 + (n-1)a_7)r(k(n-1)g(Y, U) - S(Y, U)).
\end{aligned}$$

Theorem 7.4 *Let M be an n -dimensional $(\mathcal{T}_a, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then*

$$\begin{aligned}
& \varepsilon a_5 S^2(Y, U) - E S(Y, U) - F g(Y, U) - G \eta(Y)\eta(U) \\
= & \varepsilon L(k(n-1)S^\ell(Y, U) - k^\ell(n-1)^\ell S(Y, U)),
\end{aligned}$$

where

$$E = \varepsilon(ka_0 + a_7r - k(n-1)a_1 - k(n-1)a_2),$$

$$F = -\varepsilon k(n-1)(ka_0 + k(n-1)a_4 + a_7r),$$

$$G = -k^2(n-1)^2(a_1 + a_2 + 2a_3 + a_4 + a_5 + 2a_6).$$

In view of Theorem 7.4, we have the following

Corollary 7.5 *Let M be an n -dimensional (R, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.*

fold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(Lk^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1} \right) S + kg$
Sasakian	$\left(L(n-1)^{\ell-1} - \frac{1}{n-1} \right) S + g$
Kenmotsu	$\left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1} \right) S - g$
(ε) -Sasakian	$\left(L(\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1} \right) S + \varepsilon g$
para-Sasakian	$\left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1} \right) S - g$
(ε) -para-Sasakian	$\left(L(-\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1} \right) S - \varepsilon g$

Corollary 7.6 Let M be an n -dimensional $(\mathcal{C}_*, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$L(n-1)S^\ell =$
$N(k)$ -contact metric	$-\frac{a_1}{k}S^2 - \left(\left(1 - \frac{r}{kn(n-1)} \right) a_0 - \frac{2r}{kn}a_1 - k^{\ell-1}(n-1)^\ell L \right) S$ $+(n-1) \left(\left(k - \frac{r}{n(n-1)} \right) a_0 + \left(k(n-1) - \frac{2r}{n} \right) a_1 \right) g$
Sasakian	$-a_1S^2 - \left(\left(1 - \frac{r}{n(n-1)} \right) a_0 - \frac{2r}{n}a_1 - (n-1)^\ell L \right) S$ $+(n-1) \left(\left(1 - \frac{r}{n(n-1)} \right) a_0 + \left((n-1) - \frac{2r}{n} \right) a_1 \right) g$
Kenmotsu	$a_1S^2 - \left(\left(1 + \frac{r}{n(n-1)} \right) a_0 + \frac{2r}{n}a_1 - (-1)^{\ell-1}(n-1)^\ell L \right) S$ $-(n-1) \left(\left(1 + \frac{r}{n(n-1)} \right) a_0 + \left((n-1) + \frac{2r}{n} \right) a_1 \right) g$
(ε) -Sasakian	$-\varepsilon a_1S^2 - \left(\left(1 - \frac{\varepsilon r}{n(n-1)} \right) a_0 - \frac{2\varepsilon r}{n}a_1 - (\varepsilon)^{\ell-1}(n-1)^\ell L \right) S$ $+(n-1) \left(\left(\varepsilon - \frac{r}{n(n-1)} \right) a_0 + \left(\varepsilon(n-1) - \frac{2r}{n} \right) a_1 \right) g$
para-Sasakian	$a_1S^2 - \left(\left(1 + \frac{r}{n(n-1)} \right) a_0 + \frac{2r}{n}a_1 - (-1)^{\ell-1}(n-1)^\ell L \right) S$ $-(n-1) \left(\left(1 + \frac{r}{n(n-1)} \right) a_0 + \left((n-1) + \frac{2r}{n} \right) a_1 \right) g$
(ε) -para-Sasakian	$\varepsilon a_1S^2 - \left(\left(1 + \frac{\varepsilon r}{n(n-1)} \right) a_0 + \frac{2\varepsilon r}{n}a_1 - (-\varepsilon)^{\ell-1}(n-1)^\ell L \right) S$ $-(n-1) \left(\left(\varepsilon + \frac{r}{n(n-1)} \right) a_0 + \left(\varepsilon(n-1) + \frac{2r}{n} \right) a_1 \right) g$

Corollary 7.7 Let M be an n -dimensional (\mathcal{C}, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian mani-

fold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$- \left(\frac{r}{k(n-1)^2(n-2)} + \frac{1}{n-1} - k^{\ell-1}(n-1)^{\ell-1}L \right) S$ $+ \left(k - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g + \frac{1}{k(n-1)(n-2)} S^2$
Sasakian	$- \left(\frac{r}{(n-1)^2(n-2)} + \frac{1}{n-1} - (n-1)^{\ell-1}L \right) S$ $+ \left(1 - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g + \frac{1}{(n-1)(n-2)} S^2$
Kenmotsu	$- \left(-\frac{r}{(n-1)^2(n-2)} + \frac{1}{n-1} - (-1)^{\ell-1}(n-1)^{\ell-1}L \right) S$ $+ \left(-1 - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g - \frac{1}{(n-1)(n-2)} S^2$
(ε) -Sasakian	$- \left(\frac{\varepsilon r}{(n-1)^2(n-2)} + \frac{1}{n-1} - (\varepsilon)^{\ell-1}(n-1)^{\ell-1}L \right) S$ $+ \left(\varepsilon - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g + \frac{\varepsilon}{(n-1)(n-2)} S^2$
para-Sasakian	$- \left(-\frac{r}{(n-1)^2(n-2)} + \frac{1}{n-1} - (-1)^{\ell-1}(n-1)^{\ell-1}L \right) S$ $+ \left(-1 - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g - \frac{1}{(n-1)(n-2)} S^2$
(ε) -para-Sasakian	$- \left(-\frac{\varepsilon r}{(n-1)^2(n-2)} + \frac{1}{n-1} - (-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L \right) S$ $+ \left(-\varepsilon - \frac{n-1}{n-2} + \frac{r}{(n-1)(n-2)} \right) g - \frac{\varepsilon}{(n-1)(n-2)} S^2$

Corollary 7.8 Let M be an n -dimensional (\mathcal{L}, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{k(n-1)(n-2)} S^2 + \left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1} \right) S - \frac{k}{n-2} g$
Sasakian	$\frac{1}{(n-1)(n-2)} S^2 + \left((n-1)^{\ell-1}L - \frac{1}{n-1} \right) S - \frac{1}{n-2} g$
Kenmotsu	$- \frac{1}{(n-1)(n-2)} S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1} \right) S + \frac{1}{n-2} g$
(ε) -Sasakian	$\frac{\varepsilon}{(n-1)(n-2)} S^2 + \left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1} \right) S - \frac{\varepsilon}{n-2} g$
para-Sasakian	$- \frac{1}{(n-1)(n-2)} S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1} \right) S + \frac{1}{n-2} g$
(ε) -para-Sasakian	$- \frac{\varepsilon}{(n-1)(n-2)} S^2 + \left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1} \right) S + \frac{\varepsilon}{n-2} g$

Corollary 7.9 Let M be an n -dimensional (\mathcal{V}, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian mani-

fold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$- \left(\frac{1}{n-1} - \frac{r}{kn(n-1)^2} - k^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(k - \frac{r}{n(n-1)} \right) g$
Sasakian	$- \left(\frac{1}{n-1} - \frac{r}{n(n-1)^2} - (n-1)^{\ell-1} \right) S$ $+ \left(1 - \frac{r}{n(n-1)} \right) g$
Kenmotsu	$- \left(\frac{1}{n-1} + \frac{r}{n(n-1)^2} - (-1)^{\ell-1}(n-1)^{\ell-1} \right) S$ $- \left(1 + \frac{r}{n(n-1)} \right) g$
(ε) -Sasakian	$- \left(\frac{1}{n-1} - \frac{\varepsilon r}{n(n-1)^2} - (\varepsilon)^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\varepsilon - \frac{r}{n(n-1)} \right) g$
para-Sasakian	$- \left(\frac{1}{n-1} + \frac{r}{n(n-1)^2} - (-1)^{\ell-1}(n-1)^{\ell-1} \right) S$ $- \left(1 + \frac{r}{n(n-1)} \right) g$
(ε) -para-Sasakian	$- \left(\frac{1}{n-1} + \frac{\varepsilon r}{n(n-1)^2} - (-\varepsilon)^{\ell-1}(n-1)^{\ell-1} \right) S$ $- \left(\varepsilon + \frac{r}{n(n-1)} \right) g$

Corollary 7.10 Let M be an n -dimensional $(\mathcal{P}_*, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$- \left(\left(\frac{1}{n-1} - \frac{r}{kn(n-1)^2} \right) a_0 - \frac{r}{kn(n-1)} a_1 - k^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\left(k - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$
Sasakian	$- \left(\left(\frac{1}{n-1} - \frac{r}{n(n-1)^2} \right) a_0 - \frac{r}{n(n-1)} a_1 - (n-1)^{\ell-1} \right) S$ $+ \left(\left(1 - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$
Kenmotsu	$- \left(\left(\frac{1}{n-1} + \frac{r}{n(n-1)^2} \right) a_0 + \frac{r}{n(n-1)} a_1 - (-1)^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$
(ε) -Sasakian	$- \left(\left(\frac{1}{n-1} - \frac{\varepsilon r}{n(n-1)^2} \right) a_0 - \frac{\varepsilon r}{n(n-1)} a_1 - (\varepsilon)^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\left(\varepsilon - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$
para-Sasakian	$- \left(\left(\frac{1}{n-1} + \frac{r}{n(n-1)^2} \right) a_0 + \frac{r}{n(n-1)} a_1 - (-1)^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\left(-1 - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$
(ε) -para-Sasakian	$- \left(\left(\frac{1}{n-1} + \frac{\varepsilon r}{n(n-1)^2} \right) a_0 + \frac{\varepsilon r}{n(n-1)} a_1 - (-\varepsilon)^{\ell-1}(n-1)^{\ell-1} \right) S$ $+ \left(\left(-\varepsilon - \frac{r}{n(n-1)} \right) a_0 - \frac{r}{n} a_1 \right) g$

Corollary 7.11 *Let M be an n -dimensional (\mathcal{P}, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + kg$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + g$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + \varepsilon g$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - \varepsilon g$

Corollary 7.12 *Let M be an n -dimensional (\mathcal{M}, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{2k(n-1)^2}S^2 + \left(Lk^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1}\right)S + \frac{k}{2}g$
Sasakian	$\frac{1}{2(n-1)^2}S^2 + \left(L(n-1)^{\ell-1} - \frac{1}{n-1}\right)S + \frac{1}{2}g$
Kenmotsu	$-\frac{1}{2(n-1)^2}S^2 + \left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1}\right)S - \frac{1}{2}g$
(ε) -Sasakian	$\frac{\varepsilon}{2(n-1)^2}S^2 + \left(L(\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1}\right)S + \frac{\varepsilon}{2}g$
para-Sasakian	$-\frac{1}{2(n-1)^2}S^2 + \left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1}\right)S - \frac{1}{2}g$
(ε) -para-Sasakian	$-\frac{\varepsilon}{2(n-1)^2}S^2 + \left(L(-\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{1}{n-1}\right)S - \frac{\varepsilon}{2}g$

Corollary 7.13 *Let M be an n -dimensional $(\mathcal{W}_0, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:*

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{k(n-1)^2}S^2 + \left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + kg$
Sasakian	$\frac{1}{(n-1)^2}S^2 + \left((n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + g$
Kenmotsu	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g$
(ε) -Sasakian	$\frac{\varepsilon}{(n-1)^2}S^2 + \left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + \varepsilon g$
para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g$
(ε) -para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - \varepsilon g$

Corollary 7.14 *Let M be an n -dimensional $(\mathcal{W}_0^*, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.*

ifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$-\frac{1}{k(n-1)^2}S^2 + k^{\ell-1}(n-1)^{\ell-1}LS + kg$
Sasakian	$-\frac{1}{(n-1)^2}S^2 + (n-1)^{\ell-1}LS + g$
Kenmotsu	$\frac{1}{(n-1)^2}S^2 + (-1)^{\ell-1}(n-1)^{\ell-1}LS - g$
(ε) -Sasakian	$-\frac{\varepsilon}{(n-1)^2}S^2 + (\varepsilon)^{\ell-1}(n-1)^{\ell-1}LS + \varepsilon g$
para-Sasakian	$\frac{1}{(n-1)^2}S^2 + (-1)^{\ell-1}(n-1)^{\ell-1}LS - g$
(ε) -para-Sasakian	$\frac{1}{(n-1)^2}S^2 + (-\varepsilon)^{\ell-1}(n-1)^{\ell-1}LS - \varepsilon g$

Corollary 7.15 Let M be an n -dimensional $(\mathcal{W}_1, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + kg$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + g$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + \varepsilon g$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - \varepsilon g$

Corollary 7.16 Let M be an n -dimensional $(\mathcal{W}_1^*, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + kg$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + g$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + \varepsilon g$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - \varepsilon g$

Corollary 7.17 Let M be an n -dimensional $(\mathcal{W}_2, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

ifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{k(n-1)^2}S^2 + \left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$
Sasakian	$\frac{1}{(n-1)^2}S^2 + \left((n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$
Kenmotsu	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$
(ε) -Sasakian	$\frac{\varepsilon}{(n-1)^2}S^2 + \left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$
para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$
(ε) -para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S$

Corollary 7.18 Let M be an n -dimensional (W_3, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2kg$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2g$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2g$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2\varepsilon g$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2g$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2\varepsilon g$

Corollary 7.19 Let M be an n -dimensional (W_4, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{k(n-1)^2}S^2 + \left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + kg - \varepsilon k\eta \otimes \eta$
Sasakian	$\frac{1}{(n-1)^2}S^2 + \left((n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + g - \eta \otimes \eta$
Kenmotsu	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g + \eta \otimes \eta$
(ε) -Sasakian	$\frac{\varepsilon}{(n-1)^2}S^2 + \left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S + \varepsilon g - \eta \otimes \eta$
para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - g + \eta \otimes \eta$
(ε) -para-Sasakian	$-\frac{\varepsilon}{(n-1)^2}S^2 + \left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right)S - \varepsilon g + \eta \otimes \eta$

Corollary 7.20 Let M be an n -dimensional (W_5, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

ifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\frac{1}{k(n-1)^2}S^2 + \left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + kg$
Sasakian	$\frac{1}{(n-1)^2}S^2 + \left((n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + g$
Kenmotsu	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g$
(ε) -Sasakian	$\frac{\varepsilon}{(n-1)^2}S^2 + \left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + \varepsilon g$
para-Sasakian	$-\frac{1}{(n-1)^2}S^2 + \left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g$
(ε) -para-Sasakian	$-\frac{\varepsilon}{(n-1)^2}S^2 + \left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - \varepsilon g$

Corollary 7.21 Let M be an n -dimensional $(\mathcal{W}_6, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + kg + k\eta \otimes \eta$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + g + \eta \otimes \eta$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g - \eta \otimes \eta$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + \varepsilon g + \varepsilon\eta \otimes \eta$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - g - \eta \otimes \eta$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - \varepsilon g - \varepsilon\eta \otimes \eta$

Corollary 7.22 Let M be an n -dimensional $(\mathcal{W}_7, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2kg$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2g$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2g$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S + 2\varepsilon g$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2g$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{2}{n-1}\right)S - 2\varepsilon g$

Corollary 7.23 Let M be an n -dimensional $(\mathcal{W}_8, S, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold.

ifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(Lk^{\ell-1}(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg + k\eta \otimes \eta$
Sasakian	$\left(L(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg + \eta \otimes \eta$
Kenmotsu	$\left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg - \eta \otimes \eta$
(ε) -Sasakian	$\left(L(\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg + \eta \otimes \eta$
para-Sasakian	$\left(L(-1)^{\ell-1}(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg - \eta \otimes \eta$
(ε) -para-Sasakian	$\left(L(-\varepsilon)^{\ell-1}(n-1)^{\ell-1} - \frac{2}{n-1}\right) S + kg - \eta \otimes \eta$

Corollary 7.24 Let M be an n -dimensional (W_9, S, S^ℓ) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then we have the following table:

M	$LS^\ell =$
$N(k)$ -contact metric	$\left(k^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right) S + k\eta \otimes \eta$
Sasakian	$\left((n-1)^{\ell-1}L - \frac{1}{n-1}\right) S + \eta \otimes \eta$
Kenmotsu	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right) S - \eta \otimes \eta$
(ε) -Sasakian	$\left((\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right) S + \eta \otimes \eta$
para-Sasakian	$\left((-1)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right) S - \eta \otimes \eta$
(ε) -para-Sasakian	$\left((-\varepsilon)^{\ell-1}(n-1)^{\ell-1}L - \frac{1}{n-1}\right) S - \eta \otimes \eta$

Remark 7.25 If in the Theorem 7.4, we take M be an n -dimensional (\mathcal{T}_a, S, S) -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then the result is same as given in [28, Theorem 7.6].

Corollary 7.26 Let M be an n -dimensional $(R, S_{\mathcal{T}_a}, S^\ell)$ -pseudosymmetric $(N(k), \xi)$ -semi-Riemannian manifold. Then

$$\begin{aligned}
& (a_0 + na_1 + a_2 + a_3 + a_5 + a_6) \times \\
& (Lk(n-1)S^\ell - Lk^\ell(n-1)^\ell S + kS - k^2(n-1)g) \\
= & Lr(a_4 + (n-1)a_7)(S^\ell - k^\ell(n-1)^\ell g).
\end{aligned}$$

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